Season Ticket Buyer Value and Secondary Market Options

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Abstract: Sports franchises derive significant portions of their revenues from season ticket holders. A development that may affect season ticket management is the growth of legal secondary markets. We develop a structural model that integrates both the supply and demand sides of the secondary market into season ticket buyers' ticket purchase and usage choices. We use a panel dataset that combines season and single ticket purchase records with ticket usage data to investigate the value of secondary markets. We estimate that the secondary market increases the team's season ticket revenues by about $1 million per season. At the level of the individual season ticket customer, we estimate an increase in CLV ranging from $1,327 in the lowest quality seat tier to $2,553 in the highest. In terms of value to the customer, the average dollar value of having a secondary market is $138 per season ticket. Across segments, the secondary market provides the equivalent of a 4% discount in the premium seat tier versus an 11% discount in the economy seat tier. While the secondary market creates more value in the premium-ticket tier segments, the secondary market has the most impact on behavior in the low price oriented segment.

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1. Introduction

Season ticket customers of sports franchises often exhibit strong loyalty and are an important source of team revenues. However, consumer's decisions to purchase season tickets involve a complex set of expectations and options that complicate efforts to model consumer demand. For instance, season ticket buying decisions involve significant uncertainty about product quality since customers purchase season tickets in advance of the events (Moe and Fader 2009; Desiraju and Shugan 1999; Xie and Shugan 2001). There are also issues related to bundling and quantity discounts. Consumers have the option to purchase either single game tickets at full price or season ticket packages at a discount.

A development that may have consequences for season ticket buyer management is the establishment of legal and easy to use secondary markets. While secondary markets have long been a feature of the sports industry, trusted and legal digital secondary markets such as StubHub are a relatively recent innovation. Markets like StubHub add further complexity to season ticket buyer management by creating additional options for consumers. First, a secondary market may have positive consequences on season ticket purchases since consumers can recoup costs by selling unneeded or valuable tickets. Alternatively, a secondary market might have a negative impact on season ticket sales if it creates an alternative supply of tickets that reduces the need to pre-commit to a bundle of tickets (Tuchman 2015). Furthermore, if many season ticket buyers utilize a secondary market, this may push down resale prices and make the resale option of season tickets less attractive. Whether an efficient secondary market adds value to season tickets depends on the consequences of these counteracting mechanisms. Therefore, it is not straightforward as to whether secondary markets provide an incentive or a deterrent for season ticket purchases.

The objective of our paper is to investigate how the increased options provided by a secondary market change the value proposition for potential season ticket buyers and how this change in value impacts the economic value of these customers to the team. We build a structural model of consumer demand and conduct several counter-factual analyses related to the operation and regulation of secondary markets. The structural model is developed around the idea that at the start of season, season ticket purchase decisions are based on the expected utility from planned ticket usage in terms of attendance, reselling or non-usage. It also allows sets of single game tickets to be purchased directly from the secondary market or the team to be a substitute for season ticket packages. It captures the key tradeoff involved in encouraging secondary markets. Because of the temporal separation of season ticket purchase and actual games, we allow fans to form game quality expectations based on scheduled dates, home and opponent team performance in the previous season, and team payrolls for the upcoming season. The model considers the collective utility of the entire MLB 81 game slate.

We assemble a panel data set that combines consumer ticket transactions with ticket usage records for the seasons from 2011 to 2016 for a major league baseball team. We augment the buying and usage data with secondary market listing and transaction data from a ticket broker. This provides a complete picture of season ticket holders' game level usage as we are able to observe whether each ticket was used for attendance, listed, resold, or forgone. We use actual quality preferences, based on ticket quality levels, as a source of observable heterogeneity.

A key challenge in this analysis is modeling the interdependence between the supply of and the demand for the secondary market ticket. Our research provides a contribution through the simultaneous modeling of secondary market ticket supply and demand. We also model the season ticket holder's pricing decision. Season ticket holders who decide to list a ticket for resale select a game specific listing price that maximizes the utility of attempting to resell a ticket. We derive closed form expressions for the probability of listing tickets and the differentiated optimal prices on the secondary markets that are based on game quality, secondary market demand parameters, and each seller's price sensitivity and preferences for listing tickets on the exchange. The joint model of supply and demand in the secondary market facilitates the computation of market equilibriums in the counterfactual analyses.

The overall impact of the secondary market on the attractiveness of season ticket buying involves the tradeoff between the value of the unbundling option provided by secondary markets versus the opportunity of constructing customized sets of single game tickets via the team and the secondary market. This tradeoff depends on the interdependence between supply of and demand for tickets on the secondary markets. High secondary market demand increases the value of season tickets by increasing the option value of selling unneeded or valuable tickets. In contrast, increased secondary market supply decreases the appeal of season ticket purchases because season ticket bundles may be affordably replaced with single game tickets purchased on the secondary market. Identification of the secondary market demand and supply factors is achieved through two key sources of exogenous variation in the data. First, we leverage exogenous variation in revealed team quality to identify the willingness of season ticket holders to participate in the secondary market. Second, the ease of transacting in secondary markets may change over time due to more accessible secondary market mobile apps[1] or sellers' accumulated experience in listing on secondary markets. The trend towards greater ease or convenience provides an exogenous shifter of secondary market demand.

Substantively, we find that the secondary market creates incremental value for season ticket

[1] The growth in secondary market activity is revealed by Stubhub's public financial statements. Stubhub's successfully closed transaction revenue has increased from $3,109 million in 2013 to $4,310 in 2016.

holders and thereby increases season ticket purchase and retention rates. The ability to resell tickets provides a means for season ticket buyers to benefit from unwanted or highly demanded tickets. In terms of the team's customer management metrics, our policy experiments suggest that the secondary market increases season ticket purchasing rates by 4.27 percentage points. The effect is smallest for customers that tend to choose the highest quality tickets and becomes more substantial for customers that choose lower quality tickets. We also estimate the dollar value of the secondary market by calculating the necessary discount to match the utility provided by secondary market's options. The dollar value of the secondary market varies across quality segments. For the highest quality ticket buyers, the dollar value is $160 per season ticket or about a 4% discount. For the lowest quality tickets, the secondary market provides the equivalent of 11% discount ($73 value).

When evaluating the economic value of the secondary market to season ticket revenues, we also consider the potential cannibalization of single game ticket sales due to the alternative supply of unbundled season tickets on secondary markets. If the secondary market provides a reliable source of tickets, the team may end up competing with the secondary market in terms of single game sales. We compute a conservative estimate of the impact of the secondary market on single game sales in a scenario that all season ticket holders' resale transactions replace purchases from the team. This is a conservative assumption, as it neglects the market expansion effects of the secondary market. Under this assumption, combining the revenue gains from season ticket package purchases and the potential cannibalization of single game gate ticket sales, the net revenue impact of the secondary market would be $6.1 million over 6 years.

2. Background

One key aspect of season ticket purchases is advance purchasing. Season tickets are usually purchased in advance of the season, while single game tickets are more often purchased within seasons. The marketing literature has considered the topic of advance buying with an emphasis on exploiting segment differences to maximize firm revenues (Desiraju and Shugan 1999; Xie and Shugan 2001; Moe and Fader 2009). Advance buying is especially relevant in sports since bundles of tickets are purchased before the quality of the team is fully revealed. In this type of consumer decision making, it is important to model consumer expectations. One goal of our research is to develop a modeling framework that accounts for quality expectations across a bundle of heterogeneous items. This is a complex challenge given that MLB season tickets include 81 separate elements. In the context of season ticket holder management, expectations are likely focused on the quality of the home team. Models of season ticket buying should include factors correlated with winning rates such as past success and future payrolls (Lewis 2008) and information on opponent quality.

As noted, season tickets are also distinct because they are bundles or collections of games. Previous researchers have focused on issues related to pricing bundles of tickets. Hanson and Martin (1990) formulate the bundle pricing problem as a mixed integer programing problem and investigate a variety of scenarios related to customer reservation prices, firm costs and number of components. Venkatesh and Mahajan (1993) proposed a method for optimally pricing bundles of performances based on customer's time availability to attend events and reservation prices for musical performances. Ansari et al. (1996) extend Venkatesh and Mahajan's model to also consider decisions regarding the number of events (components) to be held and for alternative objectives such as maximizing attendance. In these models, the primary focus is on the quantity discount aspect of season tickets. The key insight in this stream of research is that the valuation of the bundle should consider the cumulative value of the component parts.

The sports context includes a number of elements that complicate the analysis of bundling.

First, organizations often pursue a mixed strategy where tickets may be purchased through season ticket packages or as single game tickets. This means that the analysis needs to consider the substitutability of subsets of single game tickets for season ticket packages. Second, there has been little discussion in the literature of contexts where product quality of the bundled components is uncertain. This type of structure complicates model development as it becomes necessary to consider the role of consumer expectations. Third, the existence of a secondary market may provide a means for consumers to unbundle sets of products.

There is an established literature focused on how secondary markets in durable goods categories act as a competitor to firms' marketing efforts (Desai and Purohit 1998; Hendel and Lizzeri 1999). In the sports context, there is a limited literature focused on secondary ticket markets. This literature has mainly focused on the topic of dynamic pricing. Using data from the 2007 major league baseball season, Sweeting (2012) finds that secondary market sellers cut prices by 40 percent as time to event decreased. Zhu (2014) presents an aggregate structural model of consumer ticket purchase decisions of buying from StubHub versus teams. Zhu finds that optimal dynamic pricing by a team only results in an increase in revenue of 3.67%.

Leslie and Sorensen (2014) examine ticket resale markets for single events. They focus on the welfare implications of ticket resale markets using data on rock concerts. However, they do not explicitly model the interdependence between resale prices and listing. This is an important omission because as resale prices increase there is likely to be an endogenous increase in the utility of listing, which may increase the utility of buying season ticket packages. We propose a structural model to simultaneously consider ticket supply and aggregate demand in the secondary market. This structure facilitates counterfactual analyses related to the value of secondary markets to season ticket holders under different rules of market operation.

Two recent papers investigate the option value of secondary markets (Ishihara and Ching 2017; Shiller 2013). Ishihara and Ching (2017) model the role of used markets on new goods sales in the context of Japanese video games. In this model, consumers decide to purchase new, or used video games, or not purchase. Conditional on previous purchase, consumers decide to sell or not. This model assumes consumers are forward-looking in terms of expected resale value. Our context and model are different from Ishihara and Ching (2017) in key aspects. For example, season tickets are bundles of perishable items rather than a durable item such as a video game. This changes key elements of the decision related to purchase timing and requires that any modeling effort considers the option to separate the season ticket package into component parts. The common practice of discounting season tickets is also a relevant distinction. Season ticket prices are often cheaper than resale prices offered on secondary markets.

3. Data, Model Free Evidence and Reduced Form Analyses

Our data includes transaction histories for season ticket buyers of a Major League Baseball team for the seasons from 2011 to 2016. The sample consists of 1,924 customers[2] who purchased season ticket packages at least once between the seasons from 2011 to 2016. Each customer has a unique account number that allows tracking each customer's season and single game ticket purchases over the 6-year period. For each transaction, we observe the ticket type (quality tier) purchased and the prices paid. The team also tracks ticket usage through bar codes, and is able to monitor attendance and ticket resales conducted via StubHub. However, the team data contains successful resale information but does not include information on listed tickets that do not sell. We augment the team data with ticket listing data from a data broker. We are, therefore, able to observe whether a

[2] The estimation sample is a sample of the population of season ticket holders. Ticket brokers and customers for whom matching across databases was imperfect are excluded from the sample.

ticket is used for attendance, listed, resold or forgone.[3]

In this section, we provide several sets of descriptive data and reduced from analyses that provide insight into the relationship between the options afforded by the secondary market activity and customer behaviors such as retention. This material reveals basic patterns of consumer behavior and motivates the structure of our model in the next section. In these analyses we also devote significant attention to segment level differences based on ticket quality preferences. This material highlights important customer management issues faced by the team.

3.1 Descriptive Statistics

Tables 1 and 2 illustrate several issues related to consumer demand over time. Table 1 shows the distribution of ticket quality and purchase incidence decisions across seasons. The table is organized around the 6 ticket quality tiers defined by the team. The purchase incidence rate of season tickets for the sample was 66.4% during the 6-year period. The table shows the proportion of customers purchasing within each of the quality tiers and the percentage that do not buy in a given season. In terms of ticket quality, about 11% of customers purchase in the highest priced tier (Tier 1), while about 6% purchase in the lowest quality level (Tier 6). The most common quality tier for season ticket holders is Tier 2, which accounts for approximately 20% of customers. The proportion of the sample not purchasing season ticket packages increased over time, from 26% in the 2011 season to approximately 50% in the 2016 season due to declining on-field performance over the 6 years. Table 2 shows the renewal rates for season ticket buyers, conditional on previous seat tier choice. There is a substantial stickiness in the purchase of season tickets. The year-to-year renewal rate of season tickets is over 83% in the highest quality seat tiers (Tier 1-4). Renewal

[3] While the use of bar code technology and the observability of the secondary market provide unprecedented levels of ticket usage monitoring, the club's ability to monitor fan behavior is still imperfect. The team does not know if tickets are given away or sold via private transactions.

rates for Tiers 5 and 6 are about 75%. There is limited switching across seat tiers.

Table 3 shows data related to pricing and the importance of season ticket sales across sections. The top of Table 3 presents average per game season ticket prices and single game ticket prices across tiers. There are several notable aspects of the pricing schedule. *First*, ticket prices are substantially different across seat tiers. The team under study classifies tickets into 6 quality tiers. The average season ticket price per ticket ranged from $54 in the highest quality tier to $8 in the lowest quality tier. *Second*, season tickets are discounted from 35% to 50% relative to single game tickets, with a smaller discount (35%) for high quality tier tickets. There is smaller variation in season ticket prices within each category relative to single game prices because the team varies the prices of single games based on opponent and time factors (weekend, day versus night)[4]. The bottom of Table 3 shows the percentage of season ticket sales per tier. For the two highest quality seat tiers, 80% to 86% of seats are purchased by season ticket buyers. The percentage of tickets purchased by season ticket holders decreases as seat quality diminishes with less than 30% for Tiers 5 and 6. The concentration of season ticket purchases in the most expensive tiers highlights the economic importance of these customers.

3.2 Secondary Market Behaviors and Segment Level Differences

Table 4 provides insights into the ticket usage "options" provided by the secondary market. The top portion of the table shows consumers' "intended" ticket usage as of the day before each game. We infer "intentions" based on whether a ticket is listed prior to a given game. Specifically, if we observe that an individual has listed a ticket on the secondary market before the game we interpret this as an intention to resell rather than attend the game. The attendance and forgoing rates reflect

[4] Gate prices were set prior to the start of the season based on factors such as opponent quality and day of week. The team divides the 81 games in a season into six blocks based on management's judgement of opposing team appeal and schedule time factors (i.e. day versus night, weekday). The industry generally refers to this as variable pricing. There are approximately 10-20 games in each block. Prices also vary based on quality tier (Web Appendix A1).

the usage decisions for non-listed tickets. There is substantial variation in attending and reselling intentions across seat tiers. Higher quality ticket holders are more likely to plan on game attendance. For example, the intended attendance rates in Tier 1 and 2 are more than 70% while the rate for Tier 6 was only 50%. Broadly, resale listing rates tend to grow as ticket quality diminishes. The higher quality tier tickets (Tier 2) have a listing rate of 6%, while the lowest quality tier tickets have a 12% rate.

While listing may reflect consumer desires to sell, listings may fail if demand is weak or prices are set too high. If a ticket fails to sell, then the consumer makes an additional decision of whether or not to attend. The middle of Table 4 shows reselling success rates for listed tickets across seat tiers. On average, there is a 36% probability that a listed ticket sells on the secondary market. The resale rates in high quality tiers are about 10% higher than in low quality tiers. There is also variation in the contingency behaviors across different quality tiers when resale attempts fail. When a listing does not sell, there is a 68.4% chance ($\frac{47.83\%}{(47.83\%+22.10\%)}$) that Tier 1 season ticket holders choose to attend instead of forgoing the ticket versus 33% for Tier 6 ticket holders.

These differences in reselling and usage rates suggest that season ticket holders with different quality ticket preferences differ in their resale motivations. High quality tier holders are less likely to list and more likely to attend when listings fail. These high-value customers might have higher reservation values and therefore might price tickets higher. This also explains the lower listing frequency. For buyers of lower quality tickets, there seems to be less interest in attending games and greater interest in selling tickets. The “Actual” usage section of Table 4 reports the ultimate ticket usage decisions by seat tier.

Table 5 shows key pricing data including secondary market listing prices and resale transaction prices. List prices tend to be set at values close to single game ticket prices. The list-to-single price ratio ranges from about 1 for Tier 1 to 0.84 for Tier 6 tickets. Differences in response to failed reselling attempts may explain the variation in season ticket holders' secondary market pricing decisions. If a customer is more likely to use a ticket that fails to sell, then that customer may be more likely to try for a higher price. The observed data is consistent with our speculation that the 'residual' usage value of an unsuccessful resale could be a driver of the listing prices. Actual resale prices (from successful transactions) are lower than the listing prices. Secondary market tickets tend to sell at values between the season ticket and single game prices.

Another key question is whether resale prices and probabilities are a function of the aggregate level of season ticket reselling. This is important since increased secondary market use might impact equilibrium list prices, resale prices, and resale probabilities. Table 6 reports the results of three analyses that investigate this issue. The first column reports a logistic regression of secondary market ticket resale success as a function of the percentage of season ticket holders listing tickets on the secondary market. We include the list price ratio (listed prices versus single game prices), seat tier, and game quality measures as control variables. As a proxy for game quality we use the total gate ticket revenues for the game. The second regression models ticket list price ratios on the percentage of season ticket holders listing, game quality and seat tier. The third regression predicts resale price ratio using the same explanatory variables. We find a significant negative effect of secondary market competition on the dependent variables in each analysis. This suggests that the value provided by the secondary market to season ticket holders may be limited by supply factors. As a secondary market attracts more season ticket holders the increased supply can push down equilibrium prices and resale probabilities. [5]

3.3 Customer Retention

[5] In Web Appendix A2, we explore whether season ticket holders exhibit forward-looking behavior in ticket usage. We find no evidence that secondary market listing are affected by future games.

While the preceding analyses highlight the options available to customers and differences in behavior across quality based segments, they do not speak directly to purchase and retention. We next attempt to link secondary market activities and single game purchasing activity to customer retention. Table 7 highlights the substitutability of single game tickets for season ticket packages. The first row of the table reports the number of single game tickets purchased by customers that did not renew season tickets. When customers allow season tickets to lapse, they often continue to attend games. In the observation period, lapsed season ticket buyers purchased approximately 14 games directly from the team. Far fewer games are purchased on the secondary market versus directly from the team (0.12 versus 14 games). The second row shows single game buying patterns in the year prior to a season ticket purchase. On average, customers that became season ticket buyers purchased 23.48 games in the previous year. The higher single game purchases make sense as these consumers were likely becoming more interested in the team over time. In comparison, these future season ticket buyers were relatively inactive on the secondary market as this group purchased 0.23 games on StubHub. The segment of customers interested in season tickets has a strong preference for purchasing season and single game tickets from the team. These results guide our model development. First, the data shows that season ticket packages and single game tickets operate as substitutes. Second, the secondary market is not significant source for tickets for the season ticket oriented segment of consumers, but it is important to accommodate the alternative supply of tickets from the secondary market.

The impact of the secondary market on customer retention may also be viewed in terms of the potential incremental value that the secondary market provides to season ticket buyers. We next examine the relationship between efforts to recoup expenses and renewal rates. The “actual” recouping percentage is calculated as a dollar value of successful resales divided by the amount paid for season tickets. Figure 1 shows a scatter plot of actual recouping percentage and season ticket purchase frequency. The figure reveals a positive correlation between dollars generated in the secondary market and customer retention. The scatter plot also reveals that many customers do not attempt to recoup costs. Next, we estimated a panel logistic regression to investigate within-individual variation in renewal decisions versus "actual" recouping activities. In this analysis we control for individual renewal tendencies via random effects. The results in Table 8 show that the actual dollars recouped during a season has a significant positive impact on the renewal decision for the next season.

In addition to these bivariate relationships between renewal and dollars recouped, we also analyzed the link between individuals' secondary market activity including listing attempts, sales, and season ticket renewal decisions. We model season ticket holders' renewal rates as a function of ticket reselling success on the secondary market. We estimated a logistic regression of the form: $Renew_{it} = \beta_{0i} + Season_t\beta_1 + \beta_2 AttdRate_{i,t-1} + \beta_3 ListRate_{i,t-1} + \beta_4 ResaleRate_{i,t-1} + \beta_5 RPR_{i,t-1} + \beta_6 ResaleRate_{i,t-1} \times RPR_{i,t-1} + \epsilon_{it}$. The $Renew_{it}$ term indicates whether customer i purchases a season ticket package in season t, β_{oi} controls for individual random effects, $Season_t$ controls for year fixed effects, $AttdRate_{i,t-1}$ is last season's game attendance rates, $ListRate_{i,t-1}$ is last season's ticket listing percentage, $ResaleRate_{i,t-1}$ is the successful resale rate conditional on listing, and $RPR_{i,t-1}$ measures the average ratio between individual i's resale prices and gate prices in season t-1. This analysis leverages within-individual, across-season variations in renewal decisions, listing rates, and resale rates. Table 9 shows the panel logistic regression results. The first notable observation is that after controlling for attendance rate, a higher ticket listing percentage is a positive indicator for season ticket renewal. Second, resale success and listing prices matter. We find that the effect of successful resale rates is moderated by the resale price. At the average resale price ratio of 0.62, a one percent increase in resale rates will increase the renewal odds ratio by 1.3%. However, low resale prices can interact negatively with resale rates. If the resale price ratio is 0.11, a 1% increase in resale success reduces the renewal odds ratio by 5.8%.

3.4 Data Summary

The preceding descriptive statistics and reduced from analyses reveal important aspects of how a secondary market for tickets influences the behavior of season ticket customers. The data suggests that the secondary market provides options for ticket usage and provides value to season ticket buyers. Consumers have options to sell tickets, attend games or discard tickets. These options can also be conditional since some customers exercise the option to attend if a resale attempt fails. We also observe that season packages and single game tickets can serve as substitutes. These findings suggest that econometric analyses of fan buying behavior should explicitly model the complex set of options available to consumers. There is also correlational data that greater success in disposing of tickets on the secondary market is positively related to renewing. However, this analysis also suggests that if consumers are only able to obtain very low prices then renewal rates suffer. This analysis provides initial evidence of both the importance of the secondary market in providing incremental value to customers and also evidence that supply and demand forces can mitigate the value proposition. Finally, the differences in behavior across quality tiers highlights the importance of considering observable quality preference heterogeneity when implementing the model.

4. Model

In this section, we develop a structural model of season ticket purchasing, single game buying and game level usage. At the core of our model are the various consumer options regarding ticket type choice and usage, and the interdependence between resale listing decisions and resale prices on the secondary markets. The overarching logic of the modeling approach is illustrated in Figure 2.

4.1. Usage Decision of Season Tickets

Given the temporal separation of season ticket purchases relative to the actual usage of tickets, we start by modeling consumer's utility of using each ticket prior to the game day. The core of this analysis is the utility of attending a given game. Consumer i's utility from attending game g with a quality tier j season ticket in season t is given in equation (1):

$$u_{igt|j}^{A} = Q_{igt|j} + \varepsilon_{igt|j}^{A}, \quad (1)$$

where $Q_{igt|j}$ indicates consumer $i's$ perceived game quality before the game day and $\varepsilon_{igt|j}^{A}$ is an error term that follows the standard Type-I extreme value distribution. We specify game quality as $Q_{igt|j} = \beta_{1ij} + X_{gt}\beta_{2i} + W_{gt}\beta_{3i} + \xi_{1gt}$ where β_{1ij} are seat tier preferences, X_{gt} includes the set of game attributes known at the beginning of each season, and W_{gt} includes variables that only become available as a given game approaches. The set of game attributes X_{gt} include year fixed effects, game schedule type indicators (weekday, night, holiday), the opposing team's winning percentage from last season, and relative pay rates at the beginning of this season. The home team's quality level is captured through year fixed effects. The W_{gt}variables include data that is learned as the season progresses including home and opposing teams' cumulative winning percentage from the beginning of the season to game g, the absolute difference between the home and visiting team's winning percentage, the home team's current winning or losing streak, and the home team's current divisional standing measured by "games back" from the division leader. We demean the components of W_{gt}. Finally, we also include a game specific unobserved shock term ξ_{1gt}. This term captures game quality factors not included in the observed game attributes $\{X_{gt}, W_{gt}\}$, such as competition from other entertainment events. The realized shock ξ_{1gt} is observed by consumers but not the researcher. Consumer i with a tier j season tickets can also forgo game g. We normalize the mean utility of forgoing a ticket to zero in equation (2):

$$(2) \qquad u_{igt|j}^{F} = \varepsilon_{igt|j}^{F},$$

where the error term $\varepsilon_{igt|j}^{F}$ follows the standard Type-I extreme value distribution.

Next, we model the utility of listing a ticket for resale. We model listings rather than resale transactions, because not all listings are successful. The utility of listing (Equation 3) is the cost associated with listing on the secondary market plus a weighted average of revenues from a successful resale and the maximum value of attending or forgoing a game when an attempt fails.

$$(3)\ u_{igt|j}^{L} = -c_{it} + q(r_{igt|j}) \cdot (\delta\beta_{4i} \cdot r_{igt|j} \cdot GateP_{gt|j} + \varepsilon_{igt|j}^{L}) + \left(1 - q(r_{igt|j})\right) \cdot \max\{u_{igt|j}^{A}, u_{igt|j}^{F}\}.$$

In this equation c_{it} represents the costs (time and effort) incurred by consumer i when listing a ticket on the secondary market. We parameterize c_{it} as $\ln(c_{it}) = Z_{it}\rho_i = \rho_{1i} + \rho_{2i}\ln(Season_t) + \rho_{3i}\ln(CumListings_{i,t-1} + 1)$, as the online secondary markets become easier to use over time and the listing cost may differ depending on individuals' listing experience with the secondary market. We include the logarithm of season trend and the logarithm of an individual's cumulating listings in the observed attributes Z_{it}. The second component represents the utility from a successful resale, where q is the probability of a successful sale, and the expression in the parentheses is the revenue gain from the sale. The δ term captures the commission charged by the secondary market platform. We set δ to 0.90, as StubHub charges a 10 percent commission rate for sellers.[6] Parameter β_{4i} is the price coefficient that captures the marginal utility from a dollar gain for the consumer. This specification allows us to measure each seller's listing cost in dollar form as $\frac{c_{it}}{\beta_{4i}}$. We normalize the list prices by the tier-specific gate prices,

[6] StubHub charges 10 percent from sellers and another 25 percent commission from buyers. For example, if the listing price is $100. Sellers get $90 out of $100, and buyers pay $125. StubHub makes $35 from the transaction.

and model the ratio of list prices relative to gate prices, $r_{igt|j}$, as the decision variable. As noted in the data section, gate prices per seat tier $GateP_{jgt}$ are set at various levels before the start of the season based on the team's perceptions of each opponent's box office appeal and the schedule time factors (i.e. day versus night, weekday). We include an error term $\varepsilon_{igt|j}^{L}$ to reflect the unobserved utility from a successful resale. The last component of the equation reflects the utility associated with a failed resale attempt. When a season ticket holder cannot resell a ticket, he or she can still obtain utility by either attending a game or forgoing use of a ticket.

The utility of listing can be viewed in probabilistic terms. We illustrate the rationale behind resale attempts with two extreme scenarios. In the case that a listed ticket k has a zero percent chance to be sold on the secondary market ($q_{k,jgt} = 0$), and consumer i has a schedule conflict where $\max\{u_{igt|j}^{A}, u_{igt|j}^{F}\} = u_{igt|j}^{F}$, then no attempt at resale should be made as $u_{igt|j}^{L} = -c_{it} + u_{igt|j}^{F} < u_{igt|j}^{F}$. In contrast, if a listed ticket k can be sold for certainty ($q_{k,jgt} = 1$), then the ticket should be listed as long as the revenue gain $\delta\beta_{4i} \cdot r_{igt|j} \cdot GateP_{gt|j}$ exceeds the listing cost c_{it}.

4.2. Resale Probability

We model secondary market demand through the resale probability $q_{k,jgt}$ of each listed ticket k of tier quality j for game g in season t using an aggregate logit form as in Equation (4):

$$(4) \qquad q_{k,jgt} = \frac{\exp(\gamma_{1j}+\gamma_2 A_{gt}+\gamma_3 L_{jgt}-\gamma_4 r_{k,jgt}+\xi_{2gt})}{1+\exp(\gamma_{1j}+\gamma_2 A_{gt}+\gamma_3 L_{jgt}-\gamma_4 r_{k,jgt}+\xi_{2gt})},$$

where γ_{1j} are tier specific intercepts, term A_{gt} represents perceived game quality for game g an average fan, term L_{jgt} is the observed percentage of season tickets listed on secondary markets and term $r_{k,jgt}$ is the ratio of secondary market price relative to gate price. Coefficient γ_4 measures secondary market buyers' price sensitivity.

We specify the perceived game quality of an average secondary market ticket buyer as

$A_{gt} = X_{gt}\bar{\beta}_2 + W_{gt}\bar{\beta}_3$. The game quality is a function of the information available prior to the season, X_{gt}, and the team and opponent performance data revealed as a season progresses, W_{gt}. We use the average preference parameters to approximate the quality perception of an average fan. We also include an unobserved secondary market demand shock ξ_{2gt}, and allow a covariance structure between ξ_{1gt} and ξ_{2gt} where $\xi_{gt} \sim N(0, \Sigma_\xi)$. We allow correlation because the same unobserved factor could drive season ticket holders' attendance and secondary market demand for the game. For simplicity, we denote $a_{jgt} = \gamma_{1j} + \gamma_2 A_{gt} + \gamma_3 L_{jgt} + \xi_{2gt}$, and write the probability:

$$q_{k,jgt} = \frac{exp(a_{jgt} - \gamma_4 r_{k,jgt})}{1 + exp(a_{jgt} - \gamma_4 r_{k,jgt})}. \quad (5)$$

4.3. Secondary Market Listing Prices

A critical component of the model is the customer's decision of the listing price when selling on the secondary market. Given the tradeoff between recouping costs, making a sale, and the utility of attending or forgoing following a failed resale, each customer decides on a game specific list price ratio $r^*_{igt|j}$ (list price relative to gate price) that maximizes utility. An expression for a consumer's optimal list price ratio $r_{igt|j}$ can be determined by taking the first-order condition of Equation (3). The optimal list price ratio is written as:

$$r^*_{igt|j} = \frac{\max\left\{u^A_{igt|j}, u^F_{igt|j}\right\} - \varepsilon^L_{igt|j}}{\delta \times \beta_{4i} \times GateP_{gt|j}} + \frac{q\left(r^*_{igt|j}\right)}{\left|\partial q\left(r^*_{igt|j}\right) / \partial r_{igt|j}\right|}. \quad (6)$$

Depending on the shape of the secondary market demand curve $q\left(r^*_{igt|j}\right)$, there is a shared level of the optimal list price ratio $\frac{q\left(r^*_{igt|j}\right)}{\left|\partial q\left(r^*_{igt|j}\right) / \partial r_{igt|j}\right|}$. However, every individual will have a different markup $\frac{\max\left\{u^A_{igt|j}, u^F_{igt|j}\right\} - \varepsilon^L_{igt|j}}{\delta \times \beta_{4i} \times GateP_{gt|j}}$, as he/she has a different utility of attending or forgoing a game. This feature of the first order condition explains why conditional on seat tier and game attributes there is still a distribution of list prices. If we insert the resale probability $q_{k,jgt}$ from equation (4) into equation (6), we obtain a closed-form expression for the optimal list price ratio (see Web Appendix B1):

$$(7) \qquad r^*_{igt|j} = \frac{a_{jgt} - \ln t_{igt|j} + W(t_{igt|j})}{\gamma_4},$$

where $t_{igt|j} = exp\left(a_{gt|j} - \gamma_4 \times \frac{\max\{u^A_{igt|j}, u^F_{igt|j}\} - \varepsilon^L_{igt|j}}{\delta \times \beta_{4i} \times GateP_{gt|j}} - 1\right)$ and W(.) is the Lambert-W function. Given that the observed listing price ratio may be different from the analytical optimal list price, we add the random error and let $r_{igt|j} = r^*_{igt|j} + \varepsilon^r_{igt|j}$ where $\varepsilon^r_{igt|j} \sim N(0, \sigma^2_r)$.

4.4. The Probability of Listing Season Tickets to Resell

Season ticket holders will list a ticket on secondary markets if the utility of listing is larger than the maximum utility from either attending or forgoing a ticket. This can be represented as:

$$(8) \qquad \Pr\left[-c_{it} + q(r_{igt|j}) \cdot (\delta\beta_{4i} \cdot r_{igt|j} \cdot GateP_{gt} + \varepsilon^L_{i|jgt}) + \left(1 - q(r_{igt|j})\right) \cdot \max\{u^A_{igt|j}, u^F_{igt|j}\} \geq \max\{u^A_{igt|j}, u^F_{igt|j}\}\right].$$

We can then insert the optimal list price ratio $r^*_{igt|j}$ (Equation 7) to obtain the following expression:

$$(9) \qquad \Pr\left[\frac{\delta\beta_{4i} \cdot GateP_{gt|j}}{\gamma_4}\left(a_{gt|j} - 1 - \ln\frac{\gamma_4 c_{it}}{\delta\beta_{4i} GateP_{gt|j}}\right) - c_{it} + \varepsilon^L_{igt|j} \geq \max\{u^A_{igt|j}, u^F_{igt|j}\}\right].$$

Given that both the error terms in the attendance and forgo utility specifications follow Type-I extreme value distributions, we can express the distribution of the maximum of the two as $\max\{u^A_{igt|j}, u^F_{igt|j}\} = v^{AF}_{igt|j} + \varepsilon^{AF}_{igt|j}$, where $v^{AF}_{igt|j} = \exp(Q_{igt|j}) + 1.0$ and $\varepsilon^{AF}_{igt|j}$ also follows the standard Type-I extreme value distribution. Since both $\varepsilon^{AF}_{igt|j}$ and $\varepsilon^L_{igt|j}$ are distributed as standard Type-I extreme value, we can derive the probability of listing a season ticket to resell as (see Web Appendices C1 and C2 for details):

(10) $\Pr\left[u_{igt|j}^{L} \geq \max\left\{u_{igt|j}^{A}, u_{igt|j}^{F}\right\}\right] = \frac{exp\left(\phi_{igt|j}^{L}\right)}{exp\left(\phi_{igt|j}^{L}\right)+exp\left(v_{igt|j}^{AF}\right)}$, where

(11) $\phi_{igt|j}^{L} = \frac{\delta\beta_{4i}\cdot GateP_{gt|j}}{\gamma_4}\left(a_{gt|j} - 1 - \ln\frac{\gamma_4 c_{it}}{\delta\beta_{4i}GateP_{gt|j}}\right) - c_{it}.$

Equations (10) and (11) show that the probability of listing a season ticket to resell is a function of secondary market demand parameters $a_{gt|j}$ and γ_4, as well as the season ticket holder's price coefficient β_{4i} and listing cost c_{it}. This allows for more resale listings as a season ticket holder's price coefficient β_{4i} increases. Alternatively, the probability of listing will decrease if the listing cost c_{it} increases or if secondary market buyers become more price sensitive γ_4. The inclusion of this supply and demand structure is essential and particularly relevant for counterfactual analyses.

4.5. Season Ticket Purchase

The season ticket purchase decision is made prior to the start of the season. This introduces significant uncertainty as consumers can only make probabilistic judgments about team quality over the season. Our assumption is that consumers have rational expectations of game quality based on information $\{X_{gt}\}$ available before the start of the season. The expected game quality before the start of a season takes the form $\mathrm{E}(Q_{igt|j}) = \beta_{1ij} + X_{gt}\beta_{2i}$ where X_{gt} includes the set of game attributes known at the beginning of each season. In contrast to the usage decision information set, the terms for within season data W_{gt} and ξ_{1gt} are not included. An important implication of our specification of game quality is that the expected quality of a game at the time of season ticket purchase may deviate from the revealed game quality at the time of ticket usage decisions (game day). The discounts provided for season ticket packages may be viewed as compensation for the consumer's pre-commitment for this uncertainty.

The expected usage utility of any single game is a function of the three usage options, to attend, forgo, or list a ticket to resell. Before the start of a season, consumers only have the information in $\{X_{gt}\}$ to inform a season ticket purchase decision. We replace the game quality perception $Q_{igt|j}$ with $\mathrm{E}(Q_{igt|j})$ in Equation (1) and the secondary market game quality perception A_{gt} with $\mathrm{E}(A_{gt})$ in Equation (3). We denote the expected game attendance and listing utility before the start of a season as $\tilde{u}^{A}_{igt|j}$ and $\tilde{u}^{L}_{igt|j}$ where $\tilde{}$ reflects the expectation.

The expected overall usage utility for game g is the weighted average of the expected utility of attending game or forgoing a ticket, $\max\{\tilde{u}^{A}_{igt|j}, \tilde{u}^{F}_{igt|j}\}$, and the expected listing utility $\tilde{u}^{L}_{igt|j}$, with the corresponding weights equal to $\Pr\left[\max\{\tilde{u}^{A}_{igt|j}, \tilde{u}^{F}_{igt|j}\} \geq \tilde{u}^{L}_{igt|j}\right]$ and $\Pr\left[\tilde{u}^{L}_{igt|j} \geq \max\{\tilde{u}^{A}_{igt|j}, \tilde{u}^{F}_{igt|j}\}\right]$, respectively. We express the expected usage utility $USE_{igt|j}$ of a season ticket for game g, in tier j in season t for consumer i as:

$$(12)\ USE_{igt|j} = \int_{\tilde{\varepsilon}^{A}_{igt|j}} \int_{\tilde{\varepsilon}^{F}_{igt|j}} \int_{\tilde{\varepsilon}^{L}_{igt|j}} \Pr\left[\max\{\tilde{u}^{A}_{igt|j}, \tilde{u}^{F}_{igt|j}\} \geq \tilde{u}^{L}_{igt|j}\right] \cdot \max\{\tilde{u}^{A}_{igt|j}, \tilde{u}^{F}_{igt|j}\} + \Pr\left[\tilde{u}^{L}_{igt|j} \geq \max\{\tilde{u}^{A}_{igt|j}, \tilde{u}^{F}_{igt|j}\}\right] d\,\tilde{\varepsilon}^{A}_{igt|j}\, d\,\tilde{\varepsilon}^{F}_{igt|j}\, d\,\tilde{\varepsilon}^{L}_{igt|j}.$$

We show more details on the integration of error terms in Equation (19) in Web Appendix B3. Given the expected usage utility $USE_{igt|j}$, consumer i's utility of buying a season ticket follows:

$$(13) \qquad u^{S}_{ijt} = k^{S}_{i} + \tau_i \sum_{g=1}^{81} USE_{igt|j} - \tau_i \beta_{4i} SeasonP_{jt} + \varepsilon^{S}_{ijt},$$

where $SeasonP_{jt}$ refers to the season ticket price for a particular seat tier j in season t. $SeasonP_{jt}$ shares the same price coefficient β_{4i} as the resale revenue in Equation (3) as we assume the value of a dollar spent is the same as a dollar collected in the secondary market. We assume the value of the season ticket package depends on the additive sum of the usage utility from each game in a season, $\sum_{g=1}^{81} USE_{igt|j}$. We acknowledge that our additive summation assumption helps alleviate model calibration burden. Yet, it does not capture the possibility that different combinations of games attended may yield utilities beyond the pure additive summation. [7] We include a scale parameter τ_i for $\sum_{g=1}^{81} USE_{igt|j}$ and $\beta_{4i}SeasonP_{jt}$, as they involve a summation over 81 games and are on a different scale from the additive unobserved term ε_{ijt}^{S}. Intercept k_i^S captures the intrinsic value of buying a season ticket as opposed to not committing to season ticket packages.

4.6. Single Game Purchase

The model-free evidence shows that a collection of single tickets may be a substitute for a season ticket package. Rather than normalize the no-purchase option ($j = 0$) to zero, we allow the utility of not committing to season tickets to be a function of consumers buying single game tickets. We use a nested decision process with the upper level representing the choice of buying a single game ticket from the team, from the secondary market, or not at all. The lower level involves the decision of which seat quality tier to buy.

Specifically, at the lower decision level, we model the utility of buying a tier j single ticket to game g in season t directly from the team as:

$$(14) \qquad u_{ijgt}^{G} = Q_{ijgt} - \beta_{4i}GateP_{jgt} + \varepsilon_{ijgt}^{G}.$$

The game quality measure at the time of buying a gate ticket has the same specification as that in the season ticket attendance decision in Equation (1), as consumers have more revealed game quality information as the season rolls out. Similarly, we model the utility of buying a tier j single ticket to game g from secondary market as:

$$(15) \qquad u_{ijgt}^{SD} = Q_{ijgt} - \beta_{4i}SecondP_{jgt} + \varepsilon_{ijgt}^{SD}.$$

[7] Fans may derive higher attendance utility from attending a subset of games. We partially account for the interdependence across games by allowing for game preferences to be correlated in our hierarchical modeling approach. However, it is also possible that the utility of attending a subset of games may be higher than the additive utility of each game due to game complementarity beyond what is captured in the correlated game preference. Unfortunately, there is a dimensionality problem given the large number of games in a season (3^{81}=4.4e+38 ticket usage combinations in a season). Literatures on product complementarity has tended to be limited to small numbers of products and limited discrete quantities (Wales and Woodland 1983, Chiang 1991, Chintagunta 1993). Lee et al. (2013) deals with more than two products but imposes strict conditional independence assumptions.

The main difference between the equations is that in Equation (14) the consumer decision involves the gate price while in Equation (15) consumers pay the secondary market price ($SecondP_{jgt}$). We make two simplifying assumptions. First, customers are guaranteed a ticket in the secondary market[8]. Second, the secondary price, $SecondP_{jgt}$, is a fraction of the gate price $GateP_{jgt}$, where the fraction is the observed list price ratio relative to the gate prices for each game and seat tier under market equilibrium. These two assumptions allow us to incorporate the impact on season ticket buying of having secondary markets. The expanded choice options allow for the possibility that a secondary market may enhance the value of not committing to season ticket packages. We let both ε_{ijgt}^{G} and ε_{ijgt}^{SD} follow the standard Type-I extreme value distribution.

At the upper decision level, the utility of buying from the team directly, u_{igt}^{G}, or the secondary market, u_{igt}^{SD}, is given as:

$$(16) \qquad u_{igt}^{G} = k_i^G + \lambda_i \ln\left[\sum_{j=1}^{6} \exp\left(\frac{v_{ijgt}^{G}}{\lambda_i}\right)\right] + \varepsilon_{igt}^{G},$$

$$(17) \qquad u_{igt}^{SD} = k_i^{SD} + \lambda_i \ln\left[\sum_{j=1}^{6} \exp\left(\frac{v_{ijgt}^{SD}}{\lambda_i}\right)\right] + \varepsilon_{igt}^{SD},$$

where v_{ijgt}^{G} and v_{ijgt}^{SD} are the deterministic parts in the utility functions in Equations (14) and (15). The inclusive value of any seat tier choice is captured in a closed form with the scale parameter λ_i. Intercepts k_i^G and k_i^{SD} indicate the intrinsic value of choosing the specific purchase outlet as compared to the outside option of not attending a game. We normalize the mean utility of the outside option to zero such that $u_{igt}^{O} = \varepsilon_{igt}^{O}$. Terms ε_{igt}^{G}, ε_{igt}^{SD}, and ε_{igt}^{O} follow the standard Type-I extreme value distributions.

[8] The first simplifying assumptions likely results in a conservative estimated impact of the secondary market. The assumption that tickets are available reduces the appeal of buying a season ticket to obtain tickets to high demand events (Yankees, Cubs, etc…).

Next, we model the expected utility of forgoing season tickets and instead waiting to buy single game gate tickets for any subset of the 81 games at the time of season ticket purchase. We replace the game quality perception Q_{ijgt} with the expected game quality $\mathrm{E}(Q_{ijgt})$, where $\mathrm{E}(Q_{ijgt}) = \beta_{1ij} + X_{gt}\beta_{2i}$. The term X_{gt} only contains game quality information that is available before the start of the season. We also replace the secondary price $SecondP_{jgt}$ with the expected market equilibrium secondary price $\widetilde{Second}P_{jgt}$ under a rational expectation assumption. The expected secondary price is imputed via fixed point algorithms to match the calibrated listing probability with the observed season ticket holders' listing probability in the data (Equation 3). Assuming that the utility from attending games is an additive sum of the games attended[9], we write the utility of not buying a season ticket $u^S_{i,j=0,t}$ as:

$$u^S_{i0t} = \tau_i \sum_{g=1}^{81} \ln\left[1 + \exp\left(\tilde{v}^G_{igt}\right) + \exp\left(\tilde{v}^{SD}_{igt}\right)\right] + \varepsilon^S_{i0t}. \tag{18}$$

Term $\tilde{v}^G_{igt} = k^G_i + \lambda_i \ln\left[\sum_{j=1}^{6} \exp\left(\frac{\tilde{v}^G_{ijgt}}{\lambda_i}\right)\right]$ is the deterministic part of the expected utility of buying a single ticket from the team in Equation (16), and $\tilde{v}^{SD}_{igt} = k^G_i + \lambda_i \ln\left[\sum_{j=1}^{6} \exp\left(\frac{\tilde{v}^{SD}_{ijgt}}{\lambda_i}\right)\right]$ is the deterministic part of the expected utility of buying a single ticket from secondary markets in Equation (17). Consistent with the season ticket utility part, we scale the inclusive value by the scale τ_i in Equation (18). This structure captures the key tradeoff involved in having a secondary market.

4.7. Heterogeneity

We model consumer heterogeneity using a hierarchical structure. We use $\theta_i = (\beta_i, \rho_i, k_i, \lambda_i, \tau_i)'$

[9] Although the additive utility summation assumption is used to alleviate model calibration challenge, we have conducted a variety of model-free analyses related to game bundling decisions. These analyses focus on questions related to whether single game tickets are purchased simultaneously as bundles or sequentially as single games. The appendix also explores whether there are patterns in the set of games attended by consumers (Web Appendix A3). In general, there does not appear to be systematic evidence that consumers are creating their own customized bundles.

to indicate the set of parameters that vary across individuals. We include both observed and unobserved individual heterogeneity as $\theta_i = \bar{\theta} + \Pi D_i + \Sigma v_i$, where D_i is a vector of observed demographics including individual's tenure in years with the team, distance to the home stadium, and the median household income in individual's zip codes (all three on the ln scale). Σ is the variance-covariance matrix of the unobserved heterogeneity, and v_is are i.i.d. standard normal errors.

4.8. Likelihood

We let $\mathcal{L}_{it}(d_{it}, d_{igt}|\Psi, \beta_i, , \rho_i, k_i, \lambda_i, \tau_i, \gamma)$ be the likelihood of observing consumer i making season ticket purchase choice $d_{it} = \{d_{ijt}^S\}$ for season t, ticket usage, and gate ticket purchase decisions $d_{igt} = \{d_{igt|j}^A, d_{igt|j}^F, d_{igt|j}^L, d_{igt}^G, d_{ijgt}^G, d_{igt}^{SD}, d_{ijgt}^{SD}, d_{igt}^N\}$ for each game g in season t conditional on observed variables $\Psi = \{X, W, Z, SeasonP, GateP\}$. This forms the likelihood of any given path of ticket purchase and usage choices in season t as:

$$(19) \qquad \mathcal{L}_{it}(d_{it}, d_{igt}|\Psi, \beta_i, \rho_i, k_i, \lambda_i, \tau_i, \gamma) \equiv \prod_{j=1}^{6} \left[P_{ijt}^S \times \prod_g^{81} \left(P_{igt|j}^A\right)^{d_{igt|j}^A} \times \left(P_{igt|j}^F\right)^{d_{igt|j}^F} \times \left(P_{igt|j}^L\right)^{d_{igt|j}^L}\right]^{d_{ijt}^S} \times \left[P_{iot}^S \times \prod_{g=1}^{81} \prod_{j=0}^{6} \left(P_{igt}^G\right)^{d_{igt}^G} \left(P_{ijgt}^G\right)^{d_{ijgt}^G} \times \left(P_{igt}^{SD}\right)^{d_{igt}^{SD}} \left(P_{ijgt}^{SD}\right)^{d_{ijgt}^{SD}} \times \left(P_{igt}^N\right)^{d_{igt}^N}\right]^{d_{iot}^S}$$

$\mathcal{L}_{ijgt}(r_{igt|j}|d_{igt|j}^L, \Psi, \beta_i, \gamma)$ is the likelihood of consumer i listing a game g tier j season ticket at a price ratio $r_{igt|j}$ on the secondary market. Taking the product over all games in season t:

$$(20) \qquad \mathcal{L}_{it}(r_{igt|j}|d_{igt|j}^L, \Psi, \beta_i, \gamma) = \prod_{g=1}^{81} \prod_{j=1}^{6} \mathcal{L}_{ijgt}(r_{igt|j}|d_{igt|j}^L, \Psi, \beta_i, \gamma).$$

Next, we denote $\mathcal{L}_{k,jgt}(y_{k,jgt}|\Psi, \beta_i, \gamma)$to be the likelihood of a listed ticket k being sold ($y_{k,jgt} = 1$) on a secondary market for game g tier j in season t as:

$$(21) \qquad \mathcal{L}_t(y_{k,jgt}|\Psi, r_{k,jgt}, \beta_i, \gamma) = \prod_{g=1}^{81} \prod_{j=1}^{6} \prod_{k=1}^{K} {q_{k,jgt}}^{y_{k,jgt}} \times \left(1 - q_{k,jgt}\right)^{1-y_{k,jgt}}.$$

We provide details for each likelihood component in Web Appendix C. The three likelihood elements are combined to form the overall log-likelihood over T seasons.

(22) $$\mathcal{LL}\left(d_{it}, d_{igt}, r_{igt|j}, y_{k,jgt}\right) = \sum_{t=1}^{T}\sum_{i=1}^{I} \ln \mathcal{L}_{it}\left(d_{it}, d_{igt}\right) + \sum_{t=1}^{T}\sum_{i=1}^{I} \ln \mathcal{L}_{it}\left(r_{igt|j}\right) + \sum_{t=1}^{T} \mathcal{L}_t\left(y_{k,jgt}\right).$$

5. Estimation

This section describes the identification strategy, endogeneity treatments, and estimation algorithm for the model detailed above. The data required to estimate the model consists of each consumer's season ticket purchase choice $d_{it} = \{d_{ijt}^{S}\}$, ticket usage and gate ticket purchase decisions $d_{igt} = \{d_{igt|j}^{A}, d_{igt|j}^{F}, d_{igt|j}^{L}, d_{igt}^{G}, d_{ijgt}^{G}, d_{igt}^{SD}, d_{ijgt}^{SD}, d_{igt}^{N}\}$, the price ratio of each ticket listed for resale relative to the gate ticket prices $\{r_{igt|j}\}$, resale transaction records of listed resale tickets $\{y_{k,jgt}\}$, observed variables $\Psi = \{X, W, Z, SeasonP, GateP\}$, and individual specific demographics D_i. Details of the estimation procedure are provided in Web Appendix D.

5.1. Identification

The unknown parameters in the model include individual-level parameters $\{\beta_{1ij}, \beta_{2i}, \beta_{3i}, \beta_{4i}, \rho_i, \tau_i, k_i^S, k_i^G, k_i^{SD}, \lambda_i\}$, secondary market demand parameters $\gamma = \{\gamma_{1j}, \gamma_2, \gamma_3, \gamma_4\}$, the variance-covariance matrix of the demand shocks Σ_ξ, and the parameters governing the consumer heterogeneity in the hierarchical model (Π, Σ).

Parameters $\beta_{1ij}, \beta_{2i}, \beta_{3i}$ are the coefficients of seat tiers, and game attributes X_{gt} and W_{gt}, respectively. The observed pattern of varying attendance rates (relative to forgoing a game) informs tier-specific intercepts, β_{1ij}. The shape of the relationship between the game schedule X variables (i.e. weekends) and attendance rates influences the estimates of β_{2i}. Similarly, the relationship between the W variables (i.e. win loss record before game g) and attendance rates influences the estimates of β_{3i}. The game demand shocks ξ_{1gt} are essentially game level random effects. Conditional on game attributes X_{gt} and W_{gt}, games with greater attendance imply a larger ξ_{1gt}.

We next consider the market level parameters $\gamma = \{\gamma_{1j}, \gamma_2, \gamma_3, \gamma_4\}$ in the resale equation. The higher the percentage of successful resale transactions in a tier, the larger the tier intercept γ_{1j}. Coefficients $\gamma_2, \gamma_3, \gamma_4$ are associated with market-level game quality, season ticket listing percentage and the list price ratio, respectively. The exogenous variations in game attributes X_{gt} and W_{gt} shift the perceived game quality in the secondary market. The relationship between the average game quality and the successful resale identifies the coefficient γ_2. Both season ticket listing percentage and the secondary market list price ratio are endogenously determined through the season ticket holders' listing probability equation (3) and list price equation (6). The listing cost variation over time and across individuals shifts the listed ticket volume on the secondary market. Thus, the exogenous proxies (Z_{it}) for listing cost serve as an exclusion restriction for the endogeneity of listing percentage. The relationship between listed ticket volumes and resale success identifies the coefficient γ_3. Gate prices $GateP_{jgt}$ serve as the exclusion variable for the endogeneity of list price ratio. As noted the team divides the 81 games in each season into six price blocks (see Web Appendix A1). The block level gate prices are determined before the start of the season based on factors such as opponent quality and day of work. This variation in gate prices shifts the relative list price ratio. The relationship between the relative list price ratio and resale success rates influences the secondary market price coefficient γ_4.

Along with the exclusion variables, we use a data augmentation approach in the MCMC Bayesian estimation to uncover the realized demand shocks ξ_{2gt} (Yang, Chen and Allenby 2003). We treat the realizations of secondary market demand shocks ξ_{2gt} as augmented latent variables (to be drawn from the MCMC process). The augmented demand shocks ξ_{2gt} help control for the source of endogeneity. For example, a positive secondary market shock might lead to an increase in the average listing probability and list prices. The augmentation approach controls for the demand shocks and corrects the biased listing percentage and list price coefficients.

Conditional on the identification of game attribute coefficients $\{\beta_{1ij}, \beta_{2i}, \beta_{3i}\}$ and the secondary market demand parameters γ, the observed list price ratio determines β_{4i}, where a more price sensitive consumer will list the ticket at a higher price. Conditional on the identification of $\{\beta_{1ij}, \beta_{2i}, \beta_{3i}, \beta_{4i}\}$ and γ, the propensity to list a season ticket is fully determined by the listing cost c_{it} (Eq 3 and 11). The identification of listing costs c_{it} is driven by each season ticket holder's relative propensity to list a ticket as compared to forgo a ticket across games and seasons. A season ticket holder with a smaller listing cost will be more likely to list his/her ticket on the secondary market as compared to forgoing the option. The listing cost c_{it} is parameterized as $\exp(\rho_{1i} + \rho_{2i}\ln(Season_t) + \rho_{3i}\ln(CumList_{it} + 1))$. The baseline tendency of ticket listings identifies ρ_{1i}. The increase in market level listing tendency over time identifies ρ_{2i}. The within-individual listing tendency changes over time identifies ρ_{3i}.

When the consumer does not purchase season tickets, s/he faces a nested logit choice model. In the lower level, the variations in game attendance and seat tiers chosen across games with different game attributes and prices help identify the preference parameters for game quality attributes $\{\beta_{1ij}, \beta_{2i}, \beta_{3i}\}$ and the price coefficient β_{4i}.The identification of the intercepts k_i^G and k_i^{SD} in the single ticket purchase relies on the frequency of single ticket purchase from the gate versus from the secondary market versus an outside option of not attending the game. Identification of the nested logit scale parameter λ_i relies on the relationship of the inclusive value of purchasing any of the six seat tiers and the frequency of buying single ticket of game g.

The identification of the preference parameters $\{k_i^S, \tau_i\}$ at the season ticket purchase stage, relies on the temporal separation of ticket purchase and ticket usage. At the ticket purchase stage, the expected ticket usage value is exogenously determined given current available information (Equations 12 and 18). The season ticket purchase intercept k_i^S in Equation 13 is identified by season ticket purchase frequency across individuals and seasons. Identification of τ_i comes from the relationship between season ticket purchase frequency and the sum of the expected usage utility across ticket tiers and seasons. Finally, the hierarchical parameters for consumer heterogeneity are identified based on the relationship between consumer decisions and the observed demographics.

5.2. Simulation

We provide detailed simulation studies related to model identification in Web Appendix E. We simulated the season and single game ticket purchase decisions of 1,000 heterogeneous customers over multiple seasons. Conditional on season ticket purchase, we simulated game level ticket usage decisions of attending, listing or foregoing. In case of listing, we also simulated pricing decisions and reselling results. The game level shocks that enter the ticket attendance stage (equations 1, 14 and 15) and secondary market demand equation (4) are simulated as well. Overall, the simulated data structure follows the proposed model in equations (1) - (18). We are able to recover the true values within a 95% confidence interval and show that the model parameters are empirically identified. We also carry out two additional simulations with regards to two special cases. The first relates to the scale parameter τ_i that connects the ticket usage decisions and season ticket purchase decisions. We are able to recover the scale parameter even when the true value is set at zero, a special case when the expected usage utility does not influence season ticket purchase. In the second special case, we show that we are able to recover the listing cost parameter when the true listing cost is set either very low or very high.

6. Results

We present the estimation results from 100,000 MCMC iterations in Table 10. The first two blocks of results report the estimates for ticket attendance usage in Equations (1) and (14). There are several things worth noting. *First*, the magnitudes of the tier coefficients are consistent with the order of the tier choice percentage of season tickets in Table 1. *Second*, fans are more likely to attend games at night or on weekends. In terms of the game quality information, X_{gt}, available before the start of a season, we find an intuitive pattern of results. The visiting team's winning percentage last season and relative pay rates in this season are both significant drivers of attendance. *Third*, we also obtain expected coefficient signs for the game quality information W_{gt} that becomes available as the season progresses. We find that the cumulative winning percentage of the home and visiting teams are positively related to attendance. The negative sign for game competitiveness indicates that fans prefer closer matchups. Interestingly, winning and losing streaks are both related to increased attendance. The "games back" variable indicates higher attendance with higher divisional standing.

We are also able to measure each season ticket holder's listing cost in dollar form by dividing the implied listing cost by the price coefficient $\frac{\exp(c_{it})}{\exp(\beta_{4i})}$. The bottom 5%, 10%, 15% and 20% of the implied listing costs are \$3.6, \$14.4, \$34.7, and \$78.5 in season 2011 and \$2.3, \$9.1, \$31.4, \$66.9 in season 2016, respectively. Our results confirm that the average listing costs decrease over seasons (-0.12) and with more listing experience (-0.10). Thus, the option value of secondary markets for season ticket holders are increasing overtime. The estimation results also reveal the intrinsic value of buying a season ticket (Equation 13) as opposed to buying a collection of single tickets (Equation 18), where the mean estimate of the scale parameter is 0.224. In addition, we find the intercept of single ticket purchases from the secondary market (-10.61) to be much smaller than that from the gate (-4.64), indicating that gate is the predominately preferred channel for these customers. Thus, the presence of online secondary markets does not bring in significant incentives for these customers to switch away from season ticket purchases. The scale parameter in the nested logit model is small (0.13) indicating the necessity of modeling in a nested decision framework.

The bottom two blocks of Table 10 report the parameters in the demand equation for the secondary market and the demand shock variances. The estimated intercepts range from 1.095 for Tier 3 to -0.004 for Tier 6. The game quality coefficient is significantly positive (1.01). When controlling for the game quality, we find a negative relationship between list price ratio and resale probability (-3.03). We also find a negative relationship between the percentage of season tickets listed on secondary markets and the resale probability (-0.12). Controlling for endogeneity through game quality and augmented demand shocks ξ_{1gt} and ξ_{2gt} is necessary because the listing decisions are largely driven by the demand in secondary markets. If a particular game has a positive secondary market shock, this would increase the average listing probability as well as list prices. Our augmentation step controls for the unobserved demand shocks and corrects for the biases in the listing and price coefficients. In addition, the estimated correlation of the two augmented demand shocks ξ_{1gt} and ξ_{2gt} is 0.580. This provides evidence that the unobserved shock to season ticket holders' perceived game quality ξ_{1gt} and the unobserved shock to secondary market demand ξ_{2gt} are related but not identical.

Table 11 reports how observed individual heterogeneity--distance, years as a customer (tenure), and income--affects the individual level coefficients. We find that fans who live far away from the stadium are more price sensitive. Our speculation is that this result is due to idiosyncratic features of the team's history and market position. The team under study has a unique history in several respects. They were essentially the only team located in a large geographic region and the team was prominently featured in the early days of cable television. This may have created a situation where the team's fan base is more geographically dispersed than other teams. Our speculation is that distance to the stadium operates differentially based on whether or not fans are located in the team's metropolitan area. Within the metro area, we suspect that distance operates as expected with greater distances being associated with higher costs of attendance. However, for fans outside of the metro area, distance may be positively correlated with preferences. We find a lower listing cost for those who live far away and for those who have shorter tenures. Our identification strategy for listing costs ensures that we control for game attendance utility across individuals. We can separate out whether the low listing frequency is due to high listing costs, higher game attendance utility, or lower price sensitivity. Finally, the matched zip code level median income does not seem to affect most coefficients.

7. Policy Analysis

In this section we report the results from simulation studies that use the preceding models to study how the secondary market influences season ticket purchases and revenues. The specific policy experiments are motivated by the legal and marketing landscapes related to secondary markets. For example, the state of Michigan recently decriminalized the practice of selling tickets above market value (Oosting 2015). Leagues and teams are also interested in regulating secondary markets for marketing purposes. For instance, the Yankees insisted on an agreement with StubHub that prohibited resales below a minimum price (USA Today 2016). Additionally, several teams have attempted to require fans to use preferred or team owned ticket exchanges (Rovell 2015)[10].

[10] We acknowledge that our counterfactual policy analyses may be subject to Lucas critique as a "no secondary market" scenario goes beyond the range of estimation sample.

The first simulation eliminates the secondary market and quantifies the overall economic impact. The second and third scenarios set a minimum or maximum price for listed tickets on the secondary market. The fourth scenario reduces listing cost. In the simulations, we set team performance, game characteristics, and season and gate ticket prices to the levels observed in the data. The simulation predicts purchasing rates and reselling activity. These probabilities are also used to calculate estimates of 5-year customer lifetime (revenue) value and the overall impact of the secondary market. The simulation procedure is outlined in Web Appendix F.

Table 12 reports the results of these policy experiments. Considering both the positive unbundling option value and the negative secondary market cannibalization, we find that the absence of the secondary market decreases season ticket purchase rates by 4.27 percentage points. The tier level results are presented in Figure 3. Consistent with the data pattern in Table 4, we see the smallest impact for the highest quality tier and a steadily increasing effect on lower quality tiers. We further calculate that the purchase rate increase provides a $4,424,346 revenue increase over 6-years from season ticket buyers.

Table 13 reports the season ticket price decreases needed to maintain consumer utility levels if the secondary market was removed. This calculation essentially measures the value of the options provided by the secondary markets to season ticket holders. On a segment level, the secondary market provides $1.98 in value per ticket to the Tier 1 segment and $0.91 value per ticket for the Tier 6 segment. This represents a 4% price reduction in the high quality tier to more than 11% price reduction for low quality tier tickets (Tier 6).

The simulations are also useful for estimating long-term customer revenue contributions to the team. Table 14 reports the estimated customer lifetime values (5 years) and the changes in

CLV when the secondary market is eliminated.[11] CLVs are significantly different based on ticket quality tier. The average 5-year CLV in the highest quality tier is $94,799 versus only $15,456 in the lowest quality tier. The elimination of the secondary market has a significant impact across all the tiers. The largest impact is in Tier 1 (-$2,553) and the smallest impact occurs in the lowest quality tier (-$1,327). However, on a percentage basis the impact is largest for the low quality segment (-8.584%) and the smallest for the highest quality tier (-2.693%).

Extending our analyses beyond season ticket holders, we also consider the potential cannibalization of single game ticket sales on non-season ticket buyers due to the alternative supply of unbundled season tickets on secondary markets. If the secondary market provides a reliable source of tickets, the team may end up competing with the secondary market in terms of single game sales. If this occurs, then reselling activity by season ticket holders might cannibalize the single game sales of the team. While our modeling framework does not provide an explicit analysis of this type of cannibalization on single game sales, we can compute a conservative estimate of the overall impact of the secondary market on single game sales. To perform this analysis, we assume that all season ticket holders' resale transactions replace purchases from the team. This is a conservative assumption, as it neglects the market expansion effects of the secondary market. Under this assumption, the successful resale activity from season ticket holders reduces single game revenues by $763,900 over 6 years. Combining the revenue gains from season ticket package purchases and the potential cannibalization of single game gate ticket sales, the net revenue impact of the secondary market would be $3,660,446 over 6 years for the estimation sample. If we project this value to the total season ticket holder population the impact is $6.1

[11] In the discussion we use the term CLV for convenience. Our estimates are better described as customer revenue estimates. These estimates do not include revenues related to ancillary purchases or revenues generated before the 2011 season. We also assume that the marginal cost of serving a fan is zero.

million over 6 years.

The second set of simulations investigates minimum and maximum listing price policies. For example, the Yankees resisted partnering with StubHub until they were able to require a minimum price for resale tickets on StubHub. For the simulation, we set the minimum list price at half of the single gate ticket price and the maximum list price to the level of the single game ticket face value. We find that the minimum list price policy reduces season ticket purchase rates by 1.53 percentage points. This is equivalent to a $1,309,880 revenue loss from season ticket holders. Not surprisingly, the impact is the largest on low quality tier season tickets. This is also consistent with the data in Table 5 that shows that lower quality tier tickets are listed at lower price ratios on the secondary market. We find a minimal impact of capping secondary prices at the ticket face value. This may be due to the market position of the team under study. This maximum list price policy might have greater impact on teams who tend to be more constrained by capacity.

8. Discussion

Our research focuses on how secondary market options affect sports fan's preferences for purchasing season tickets. In our case, consumers have the option to directly use a ticket, resell a ticket, forgo a ticket, or purchase unbundled single game tickets from either the team or the secondary markets. These post-purchase options and decisions highlight an important aspect of our research. Academic researchers often focus on data created by transaction processing systems, while decisions related to product usage are not observable to researchers. In the case of tickets, it is increasingly possible to observe significant details related to consumption.

We find that the options created by the secondary market increase the value of purchasing ticket packages. Our results suggest that the net impact of the secondary market is to increase revenue by about $6.1 million over 6 years for the team under study. Given that sports organizations have high fixed costs, low marginal costs and perishable inventory (Cross 1997) this revenue increase may have significant implications for profit rates for some clubs. In the time-period in question, Forbes (Ozanian 2018) estimated operating income of about $15 million per year for teams located in similar markets as the focal club. This suggests that the secondary market increases profitability by about 7% per year. This is a conservative estimate, as it does not include incremental sources of revenue such as parking, concessions or merchandise. However, the importance of a result of this magnitude will be dependent on a team's market and cost structure. Across major league baseball, estimates of team's revenues vary from about $200 million for small market teams to over $600 million for teams in major markets (Forbes 2018). Estimated operating incomes range from $100 million to net losses. The importance of an incremental $1 million in revenues therefore varies by market.

Our policy experiments have important implications for teams, leagues and legislatures. We find that policies that create constraints such as minimum price floors have an adverse impact on season ticket sales. This is a complex issue since price floors may be motivated by a desire to protect brand equity. Leagues and regulators must balance these brand maintenance goals against the benefits of providing more value to teams' most valuable customers.

Our results also have implications for segment level customer management. We observe significant economic and behavioral differences based on quality of tickets purchased. One interesting aspect is that the secondary market is least impactful for buyers of the highest quality seat tier. Given the lower renewal rates for buyers of the two lowest quality seat tiers, these results suggest that increasing options and value may be a particularly useful strategy for managing more marginal customers. These types of results could be used to refine pricing policies or to devise segment level promotions. Our calculations of the equivalent discount value provided by the secondary market are an example of this type of analysis.

Our findings should be interpreted based on limitations inherent to our data. For example, while we are able to observe significant post-purchase activities we do not have complete transparency. Season ticket holders may also distribute tickets through more informal markets such as selling directly to friends or giving away tickets to family members. These types of informal transfers provide an additional option value to consumers. These options existed prior to the creation of the secondary market. It is an open research question as to how the decision to use these type of informal markets or to gift tickets is influenced by the secondary market.

Our current model framework includes significant complexity in order to account for reselling decisions, secondary market pricing decisions, demand and supply factors, and the substitutability of single game bundles for season tickets. The value of incorporating these factors is that the model can speak to a number of issues related to the impact of secondary markets existence and to evaluate potential market restrictions. However, given the complexity of the model we chose not to include several relevant aspects of the season ticket market. First, we focus only on full season packages. In practice teams may offer a wide variety of packages such as half season or customer selected bundles. Extensions to incorporate alternative packages may require further modeling efforts under our framework. For example, future research could consider multiple discreteness choice models that allow customers to simultaneously “cherry-pick” single games along with a smaller package of fixed games. Quantity aspects could also be included with multiple discreteness choices.

Second, we have also assumed that the value of a dollar is the same when consumers are buying and selling tickets. This decision was made to facilitate model estimation. However, it is entirely possible that consumers may have different price sensitivities when buying versus selling (Kahneman, Knetsch and Thaler 1991).

Furthermore, while our data includes multiple years of data for a large number of consumers, the data is sourced from a single team. Teams vary in terms of local support and on-field performance. While the direction of the findings related to the "option value" provided by the secondary market are likely robust, the magnitude of effects may change based on underlying demand levels across markets. For example, if a team has frequent sellouts then the value provided by the secondary market may be even greater if fans can frequently sell tickets above face value. The issue of demand constraints also highlights a possible modeling extension. For teams with significant capacity constraints, expectations of ticket availability may become more salient.

One limitation of our study that suggests an avenue for future research is the single-category nature of our study. While we study the sports category, secondary ticket markets also do significant business in performing arts categories. Our basic modeling structure is largely applicable to non-sports contexts in that packages are purchased based on expected value and expected resale possibilities. However, there are likely some salient different in performing arts categories relative to sports. For example, while in sports contexts events may be differentiated based on opponents, the product might be viewed as largely similar. In contrast, a theater organization might offer very different types of plays and collections of actors across a season. It might also be more difficult for consumers to form expectations about performance quality since there is a lack of objective data such as winning rates and payrolls.

There are significant opportunities for future research. For example, the dynamic nature of these secondary markets can provide information about willingness to pay for different types of tickets. One area for future research might be to focus on how the secondary market information can inform team's pricing decision. It is also possible that as consumers become more familiar with the secondary market in terms of usage and in terms of expectations of selling and buying opportunities that consumer behavior might evolve over time. However, given that we have data from only a single team it is difficult to separate out team quality factors from time trends. A dataset that includes multiple teams and consumer experiences in the market would be needed to study the full impact of consumer learning. Finally, there are contexts related to season tickets that would call for a dynamic programming model. If a club had a waiting list or a seniority system, the consumer's renewal decision would need to consider the long-term benefits of buying tickets in terms of gaining access to better tickets and that cancellations might limit buying opportunities in subsequent years.

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Tables

Table 1: Season Ticket Tier Choices % by Season

	Season 2011	Season 2012	Season 2013	Season 2014	Season 2015	Season 2016
Tier 1	12.47	11.54	10.97	11.64	11.07	7.95
Tier 2	21.21	20.06	20.01	19.59	18.40	14.66
Tier 3	13.05	11.49	11.02	11.07	10.45	8.75
Tier 4	11.22	10.55	11.28	11.80	11.17	8.94
Tier 5	9.15	9.56	9.36	9.62	7.54	5.72
Tier 6	6.60	7.28	7.07	6.44	5.41	4.26
No Purchase	26.30	29.52	30.30	29.83	35.97	49.74

Table 2: Season Ticket Renewal Rate% by Tier

last season current season	No Purchase	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	Tier 6
No Purchase	79.40	14.23	11.10	15.48	15.58	22.30	23.93
Tier 1	2.39	84.23	0.73	0.18	0.19	0.00	0.00
Tier 2	3.28	0.90	87.43	0.46	0.19	0.00	0.16
Tier 3	2.74	0.36	0.31	83.42	0.37	0.57	0.00
Tier 4	4.11	0.27	0.31	0.27	82.93	0.80	0.16
Tier 5	4.58	0.00	0.00	0.09	0.65	75.86	0.32
Tier 6	3.49	0.00	0.10	0.09	0.09	0.46	75.44

Table 3: Season and Single Ticket Price ($) per Game by Tier

	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	Tier 6
Season Ticket Price per Game ($)	54.30	46.14	31.80	22.33	12.47	8.19
	(1.97)	(1.25)	(1.32)	(3.12)	(0.60)	(0.43)
Gate Ticket Price per Game ($)	85.18	75.92	52.24	41.31	29.86	17.92
	(13.78)	(13.00)	(11.45)	(9.94)	(8.25)	(5.48)
Percentage of Sales						
% of Season Tickets	86.7	83.5	66.0	60.2	30.8	25.1
% of Non-Season Tickets	13.3	16.5	34.0	39.8	69.2	74.9

Note: (1) standard deviations in parentheses; (2) variations of season ticket prices come from across seasons, while variations in single gate ticket price come from both across seasons and across games within a season.

Table 4: Season Ticket Intended and Actual Usage Patterns by Tier (in %)

	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	Tier 6
Intended Usage						
Attendance	71.21	70.31	68.34	66.02	57.54	50.42
Forgo	21.80	23.86	25.03	29.78	34.53	37.59
Listing	7.00	5.82	6.62	4.19	7.92	11.99
Conditional Usage						
Successful Resale Rates	30.07	31.26	30.14	39.04	41.06	41.12
Not Sold but Choose to Attend	47.83	46.98	49.70	33.33	25.86	19.39
Not Sold but Choose to Forgo	22.10	21.77	20.17	27.62	33.08	39.49
Actual Usage						
Attendance	74.55	73.05	71.64	67.42	59.59	52.75
Forgo	23.34	25.14	26.37	30.94	37.16	42.32
Resold	2.10	1.82	2.00	1.64	3.25	4.93

Table 5: Ticket List and Resale Transaction Price ($) per Game by Tier

	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	Tier 6
List Price per Game	83.08	72.79	49.51	38.37	27.09	15.16
	(21.86)	(22.28)	(17.32)	(15.32)	(12.30)	(8.32)
Resale Price per Game	54.65	51.47	34.31	27.15	18.42	9.93
	(18.43)	(18.64)	(13.92)	(12.80)	(9.90)	(6.30)
Season-to-Single Price Ratio	0.65	0.63	0.64	0.57	0.46	0.51
	(0.11)	(0.11)	(0.14)	(0.16)	(0.14)	(0.19)
List-to-Single Price Ratio	0.97	0.95	0.94	0.92	0.89	0.84
	(0.17)	(0.19)	(0.20)	(0.23)	(0.25)	(0.31)
Resale -to-Single Price Ratio	0.64	0.67	0.65	0.64	0.60	0.55
	(0.16)	(0.17)	(0.18)	(0.24)	(0.25)	(0.32)

Note: standard deviations in parentheses; the variation in list and resale prices comes from across seasons, across games within a season, as well as across individuals.

Table 6: Secondary Market Prices and Sales

	Logistic Regression on Resale			Linear Regression on Listing Price Ratio			Linear Regression on Resale Price Ratio		
	Estimate	S.E.		Estimate	S.E.		Estimate	S.E.	
Game Quality Index	0.720	0.012	***	0.067	0.008	***	0.127	0.008	***
% of Listing (ln)	-0.182	0.025	***	-0.023	0.005	***	-0.073	0.008	***
List Price Ratio	-3.238	0.033	***						
Seat Tier Dummies		Included			Included			Included	
Game Random Effect					Included			Included	
# of Observations	41,681			41,681			14,496		
R-Squared				0.071			0.072		

Table 7: Average Number of Games Purchased When Not Purchasing Season Tickets

	Average # Games Purchased via Primary Market	Average # Games Purchased via Online Secondary Market
In the year not renewing a season ticket	14.20	0.12
In the year prior to a season ticket purchase	23.48	0.23

Table 8: Panel Logistic Regression of Season Ticket Renewal and Secondary Market Usage

	Estimate	S.E.	T value	p-value	
ActualRecoupDollar% Last Season	0.107	0.047	2.296	<0.05	**
s.d. β_{oi}	0.326	0.006	50.238	<0.01	***
s.d. ϵ_{it}	0.349	0.003	126.394	<0.01	***

Note: ***, **, * indicates p-value <0.01, <0.05, and <0.1.

Table 9: Panel Logistic Regression of Season Ticket Renewal and Secondary Markets

	Estimate	S.E.	T value	p-value	
AttdRate	0.688	0.015	45.542	<0.01	***
ListRate	0.152	0.032	4.717	<0.01	***
ResaleRate	-0.075	0.032	4.717	<0.01	***
ResalePriceRatio	-0.022	0.025	-0.885	0.376	
ResaleRate×ResalePriceRatio	0.141	0.078	1.800	0.072	*
s.d. β_{oi}	0.156	0.006	24.913	<0.01	***
s.d. ϵ_{it}	0.343	0.003	121.763	<0.01	***
Season Dummies	Included				

Note: ***, **, * indicates p-value <0.01, <0.05, and <0.1.

Table 10: Estimation Results

	Estimate	2.5 Percentile	97.5 Percentile
Game attendance variables X_{gt} available at season ticket purchase stage			
Tier 1	-2.946	-3.092	-2.793
Difference between Tier 1 and Tier 2	0.394	0.332	0.453
Difference between Tier 1 and Tier 3	-0.320	-0.400	-0.215
Difference between Tier 1 and Tier 4	-0.811	-0.913	-0.728
Difference between Tier 1 and Tier 5	-1.407	-1.518	-1.323
Difference between Tier 1 and Tier 6	-3.250	-3.347	-3.159
Season 2012	0.162	0.126	0.205
Season 2013	0.181	0.145	0.226
Season 2014	0.091	0.050	0.138

Season 2015	-0.374	-0.418	-0.330
Season 2016	-0.628	-0.705	-0.569
Weekend	0.627	0.612	0.641
Night	0.429	0.414	0.445
Holiday	0.117	0.093	0.138
OppWin% (t-1)	2.298	2.072	2.530
OppRelPay (t)	0.334	0.316	0.356
Game attendance variables W_{gt} available at ticket usage stage			
HomeCumWinPt (gt)	0.451	0.365	0.541
OppCumWinPt (gt)	1.774	1.543	1.967
Competitiveness (gt)	-0.503	-0.622	-0.398
StreatkWin (gt)	0.029	0.024	0.034
StreakLoss (gt)	0.021	0.015	0.028
GoBack (gt)	-0.288	-0.301	-0.274
Game listing variables			
Listing Cost Intercept ρ_1	1.944	1.856	2.032
Cost Time trend ρ_2	-0.118	-0.179	-0.049
Cost Cumulative Listing ρ_3	-0.102	-0.119	-0.073
Season Ticket Price Coefficient (ln scale)	0.403	0.377	0.432
Ticket purchase intercept and scale			
Season Ticket Intercept	-2.146	-2.291	-1.983
Single Gate Ticket Intercept	-4.640	-4.755	-4.563
Single Secondary Ticket Intercept	-10.608	-10.954	-10.041
81 Games Scale τ_i (logit scale)	-1.242	-1.327	-1.116
Nested logit scale (logit scale)	-1.926	-2.010	-1.846
Secondary Market Parameters			
Tier1	0.928	0.614	1.247
Difference between Tier 1 and Tier 2	0.105	0.001	0.215
Difference between Tier 1 and Tier 3	0.167	0.067	0.275
Difference between Tier 1 and Tier 4	-0.056	-0.167	0.059
Difference between Tier 1 and Tier 5	-0.289	-0.382	-0.184
Difference between Tier 1 and Tier 6	-0.932	-1.037	-0.815
Quality Coefficient A_{gt}	1.008	0.877	1.128
% of Season Tickets Listed L_{jgt}	-0.121	-0.207	-0.051
Price Coefficient of Secondary Market (ln scale)	1.117	1.091	1.142
Variance of ξ_{1gt}	0.198	0.179	0.227
Variance of ξ_{2gt}	1.710	1.479	1.958
Cor between the two shocks ξ_{1gt} and ξ_{2gt}	0.580	0.550	0.611
Error variance in the list price	0.183	0.160	0.213
Log-likelihood	-621,560		

Note: (1) we ran a total of 100,000 MCMC iterations and report the posterior distributions of the parameters based on the last 20,000 iterations; (2) the model has an 89.06% hit rate for the season ticket purchase decisions with the hit rates of season ticket tier choices range from 79% to 95%; (3) price coefficients are reparameterized as -exp(.), cost coefficients are reparametrized as -exp(.), and scale coefficient λ and τ are reparameterized as $\frac{\exp(\cdot)}{1+\exp(\cdot)}$; (3) in the estimation we use Tier 1 as the baseline and estimate the differences between the other five tiers and Tier 1. This allows the Bayesian MCMC algorithm to be more efficient in convergence.

Table 11: Observed Individual Heterogeneity

	Demeaned Distance	Demeaned Tenure	Demeaned Income
Tier 1	0.081	**1.423**	-0.075
Difference between Tier 1 and Tier 2	-0.049	0.140	0.154
Difference between Tier 1 and Tier 3	**0.357**	0.028	-0.020
Difference between Tier 1 and Tier 4	0.104	-0.184	-0.229
Difference between Tier 1 and Tier 5	**0.400**	**-0.677**	**-0.798**
Difference between Tier 1 and Tier 6	-0.009	-0.265	0.081
Season 2012	**0.099**	**-0.481**	**-0.177**
Season 2013	0.010	**-1.307**	-0.010
Season 2014	-0.097	**-1.402**	0.056
Season 2015	**-0.119**	**-1.517**	0.151
Season 2016	**-0.151**	**-1.264**	0.102
Weekend	**0.093**	**-0.084**	**-0.145**
Night	**-0.033**	**0.049**	**0.068**
Holiday	**0.047**	0.016	-0.057
OppWin% (t-1)	**-0.293**	**0.993**	0.199
OppRelPay (t)	0.005	**-0.037**	0.028
HomeCumWinPt (gt)	0.028	**0.123**	-0.020
OppCumWinPt (gt)	**-0.258**	**0.695**	-0.055
Competitiveness (gt)	-0.038	-0.156	-0.255
StreatkWin (gt)	0.001	0.010	-0.003
StreakLoss (gt)	0.005	-0.001	-0.007
GoBack (gt)	-0.004	0.007	-0.006
Listing Cost Intercept	**-0.498**	**0.247**	-0.309
Cost Time Trend (ln)	**0.236**	-0.133	0.056
Cost Cumulative Listings (ln)	**-0.115**	**0.171**	-0.094
Price Coefficient of Season Ticket Holders	**0.197**	**-0.223**	-0.019
Season Ticket Intercept	-0.072	**0.925**	-0.036
Single Gate Ticket Intercept	**0.232**	-0.251	**0.556**
Single Secondary Ticket Intercept	0.091	-0.309	0.077
81 Games Scale	**-0.222**	**1.203**	**-0.375**
Nested Logit Scale	-0.012	**0.208**	0.084

Note: bold refers to 95% credit interval does not cover zero.

Table 12: Policy Experiments

	Season Ticket Purchase Rate Difference	Season Ticket Purchase Rate % Change	Revenue Change ($)
No secondary market	-4.27%	-6.45%	-4,424,346
Minimum list price policy	-1.53%	-2.32%	-1,309,880
Maximum list price policy	-0.15%	-0.23%	-350,639
Listing cost reduced to 50%	1.63%	2.46%	1,092,011

Note: the rate difference refers to purchase rate percentage point differences in the counterfactual setting and the baseline prediction setting, while the rate % change refers to the percentage changes.

Table 13: Certainty Equivalence Season Ticket Price Changes without Secondary Markets

	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	Tier 6
Certainty Equivalence ($) in Season Prices	160.34	196.59	133.69	118.48	57.73	73.87
Certainty Equivalence ($) per Season Game	1.98	2.43	1.65	1.46	0.71	0.91
Certainty Equivalence out of Season Price %	3.65%	5.26%	5.19%	6.55%	5.72%	11.14%

Note: certainty equivalence refers to how much season price drop is needed to keep the usage utility equivalent when secondary markets do not exist.

Table 14: Customer Lifetime Value with/without Secondary Markets

	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	Tier 6
CLV ($) with Secondary Market	94,799	84,163	56,757	40,608	22,680	15,456
CLV changes ($) without Secondary Market	-2,553	-2,280	-2,039	-1,528	-1,490	-1,327
CLV change %	-2.69%	-2.71%	-3.39%	-3.76%	-6.57%	-8.58%

Figures:

Figure 1: Intended Usage Percentage and Retention Rate

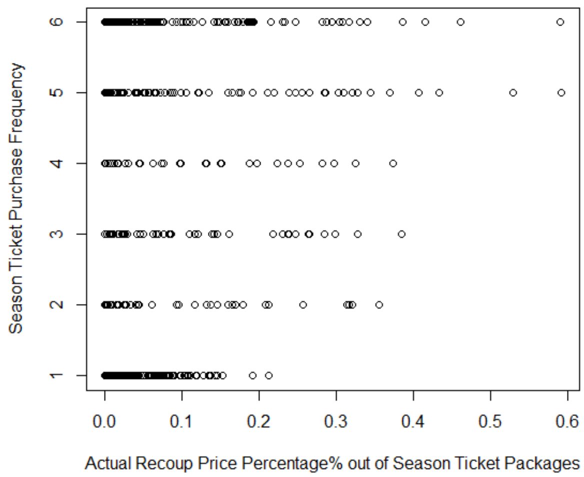


Figure 2: Illustration of the Decision Process

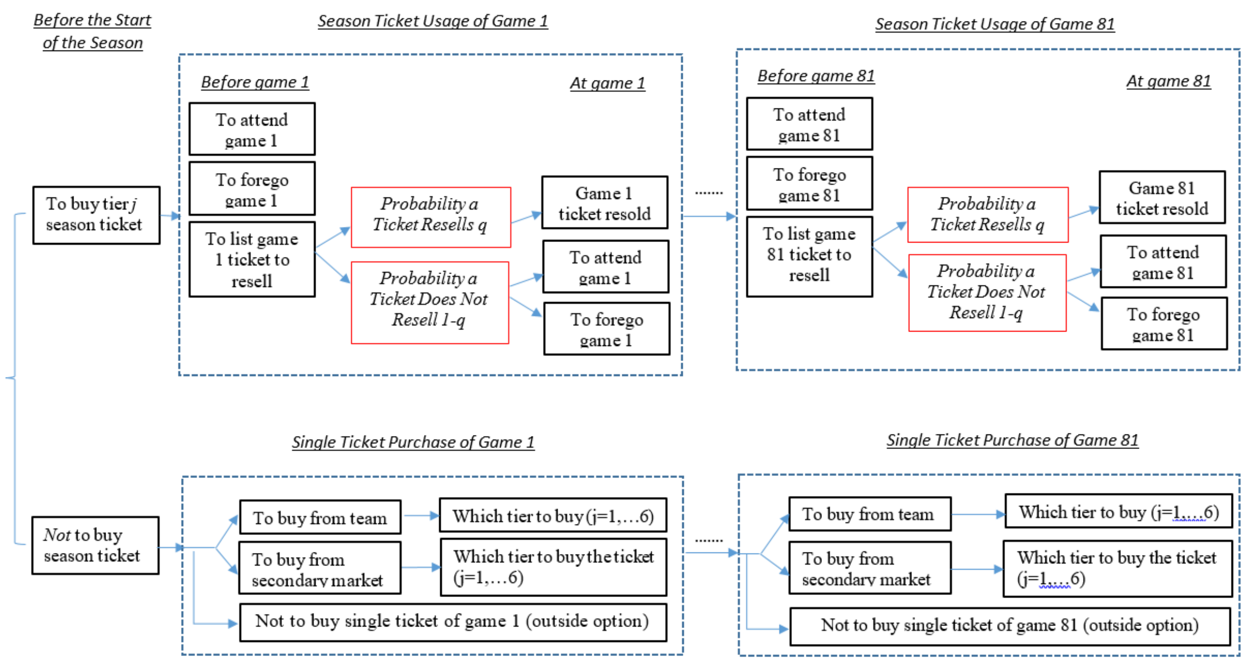
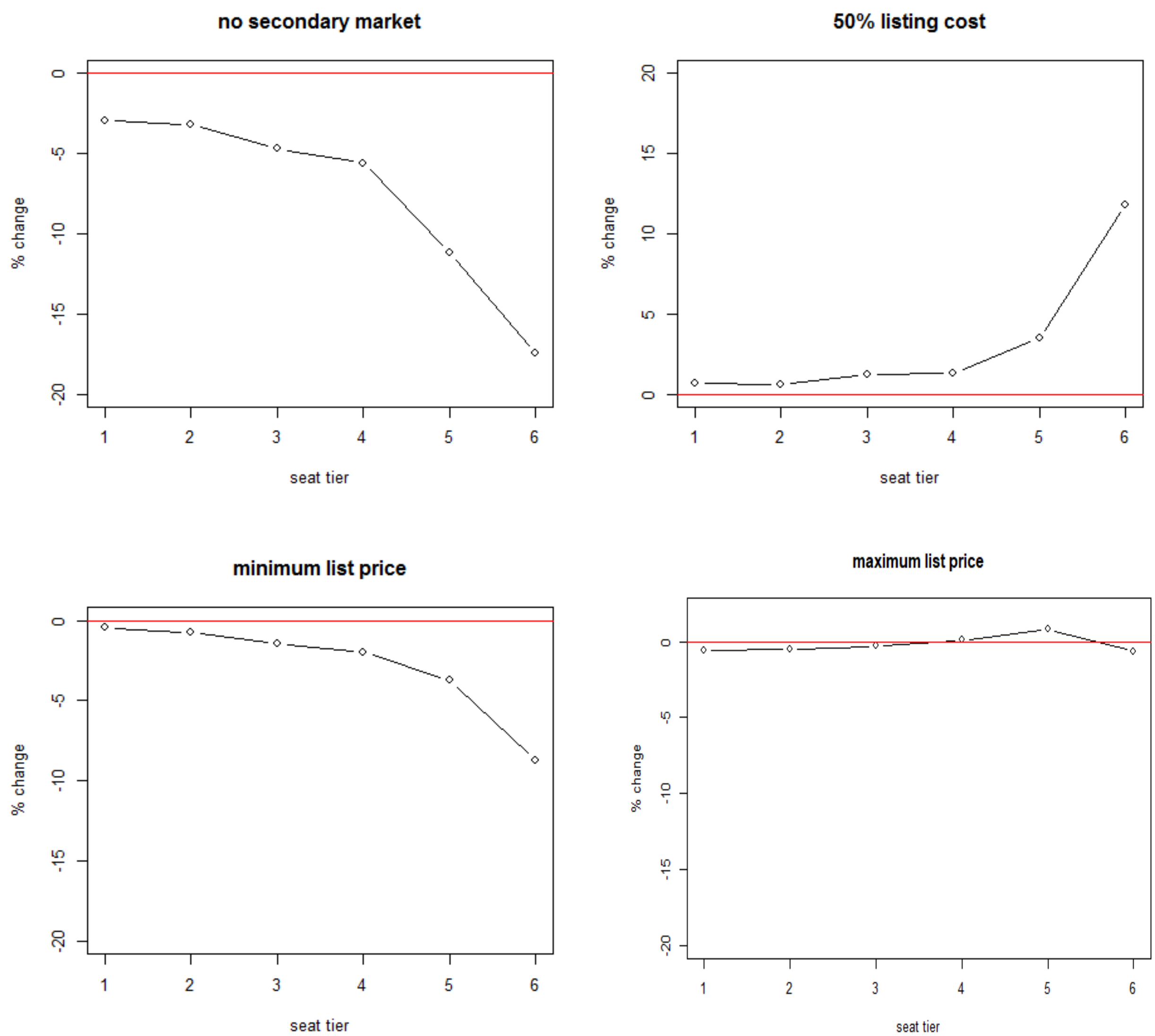


Figure 3: Policy Experiment Results by Tier



Web Appendices

A Additional Model-Free Evidence

A.1 Gate Price Variations

Gate prices were set prior to the start of the season based on factors such as opponent quality and day of week. The team divides the 81 games in a season into six blocks based on management's judgement of opposing team appeal and schedule time factors (i.e. day versus night, weekday). The industry generally refers to this as variable pricing. There are approximately 10-20 games in each block. Prices also vary based on quality tier. There are six quality tiers. The Figure below illustrates the gate price per game in the 2012 season in each seat tier and game block. It varies from \$112 per game in game Block 1 and seat Tier 1 to \$10 per game in game Block 6 and seat tier 6.

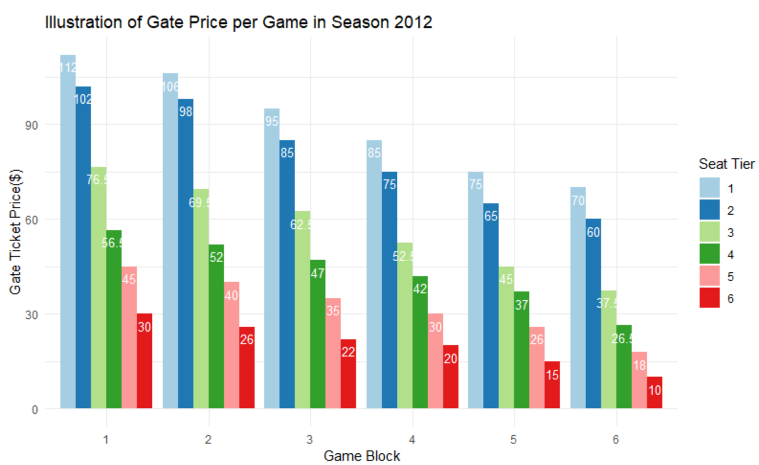


Figure 1: Illustration of Gate Price per Game in Season 2012

The gate price per game increased slightly over the years. The black bars in the Figure below indicate the average gate price per game over the six years.

Table A1 shows the summary statistics regarding game attendance in different game blocks for Year 2012. It shows that there is great variation in game attendance within each game block. The prices determined before the start of the season do not explain all the variations in game appeal during the season. An ANOVA procedure reveals that controlling

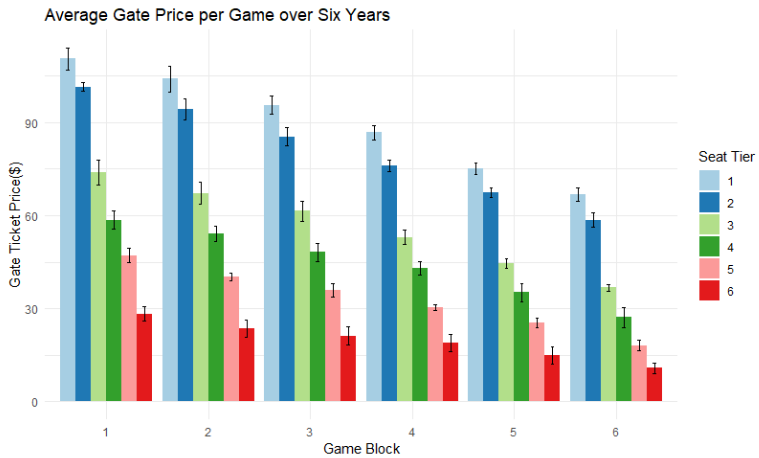


Figure 2: Average Gate Price per Game over Six Years

for season fixed effects and game block fixed effects, the variance in game attendance within each game block accounts for 79% of the overall game attendance variance. The implication is that there is still considerable variance due to differences in opponent quality (both permanent and dynamically revealed during each season) and other game traits. This variation provides an exogenous factor that helps identify the secondary market supply parameters.

Given that the gate prices were not dynamically set during the season based on the within season results and demand levels, we consider the gate price as an exogenous variable during a season. The substantial variations in game attendance within each game block also provides evidence for the exogeneity of gate prices.

Table A1: Game Attendances Across Game Blocks for Season 2012

Game Block	N Games	Attendance				
		Mean	Median	SD	Min	Max
1	11	35,199	37,421	7,922	20,947	47,510
2	7	36,573	36,261	3,688	31,230	41,833
3	8	30,173	31,469	5,256	19,871	35,649
4	21	17,096	17,435	5,704	7,925	33,708
5	20	26,407	25,006	9,464	12,462	48,060
6	14	17,761	17,259	5,463	10,735	30,022

A.2 Forward-looking Behavior in Ticket Usage

An open question is whether consumers make ticket usage and selling decisions strategically across groups of games. If this is the case, it might be necessary to model sequences of ticket usage decisions rather than game level decisions. As a test of whether fans make decisions across sequences of games, we estimate a logistic regression of season ticket holder's game level resale listing decisions (yes=1, no=0) on the game quality index of the current and the next five games. We again use individual game ticket sales revenue as an approximation for game quality and include quality tier level fixed effects. We also control for cross-individual variations in individual listing rates, as well as non-usage rates. Table A2 shows that the coefficients of game quality index of the future five games are all non-significant, while current game quality has a significant positive coefficient. It suggests that season ticket holders are more likely to list tickets for an attractive game, and that they do not make listing decisions based on future games.

Table A2: Game Level Listing Decision and the Possibilities of Forward Looking

	Estimate	S.E.	t-value	p-value	
Current period game quality index	0.049	0.012	3.909	<0.001	***
Future period game quality index (g+1)	0.019	0.015	1.286	0.199	
Future period game quality index (g+2)	-0.012	0.015	-0.787	0.431	
Future period game quality index (g+3)	0.013	0.015	0.878	0.379	
Future period game quality index (g+4)	-0.011	0.015	-0.695	0.487	
Future period game quality index (g+5)	-0.007	0.013	-0.527	0.598	
Individual average listing rate	6.179	0.033	190.765	<0.001	***
Individual average forgo rate	0.173	0.053	3.236	<0.001	***
Seat tier dummies	Included				
# of observations	188,976				

Note: (1) consumer-game level listing decision (yes=1/no=0) is the dependent variable; (2) we control for cross-individual variation with individual average listing rate and forgo rate in each season; (3) we use the logarithm of total revenue from gate ticket sales as an approximation for game quality index.

A.3 Bundling Issues

Another necessary assumption we impose is that the utility of not purchasing a season ticket is an additive sum of the purchased single games. We acknowledge that this additive form assumption does not capture the possibility that different combinations of single games may yield utilities beyond the pure summation (i.e. bundling effect). This assumption helps maintain model tractability. In addition, we show empirically that when not committing to season ticket packages, customers tend to buy single tickets at different points of time rather than buying a select subset of games altogether. We view this as partial evidence that the true purchase decision process involves independent decisions on each game.

First, we examine whether consumers tend to purchase games in bundles when not committing to season ticket packages. In particular, we focus on whether consumers would purchase different combinations of games. If consumers would receive more utility from certain combinations of games, we suspect that they may tend to purchase the particular combinations of games at the same time. Figure (a) (left) above shows a histogram of the number of single game tickets purchased when consumers did not commit to season ticket packages. In general, consumers purchase significantly fewer tickets when not committing to season ticket packages. Figure (b) (right) shows the proportion of games that were purchased altogether as a combination. The x-axis indicates the number of games in the purchase bundle, while the y-axis indicates the proportion. As can be seen, approximately 67% of games were purchased separately, while 14% of games were purchased along with one other (two games purchased at the same time). While consumers do not necessarily need to purchase games that provide some type of joint benefit simultaneously we suspect that consumer customized bundles would tend to be purchased in the same transaction. However, the empirical data pattern provides some support for our assumption that consumers purchase single game tickets independently.

We also plot the distribution of purchase times versus purchased games in Figure 4. The most common dominant purchase pattern is to purchase each game independently regardless

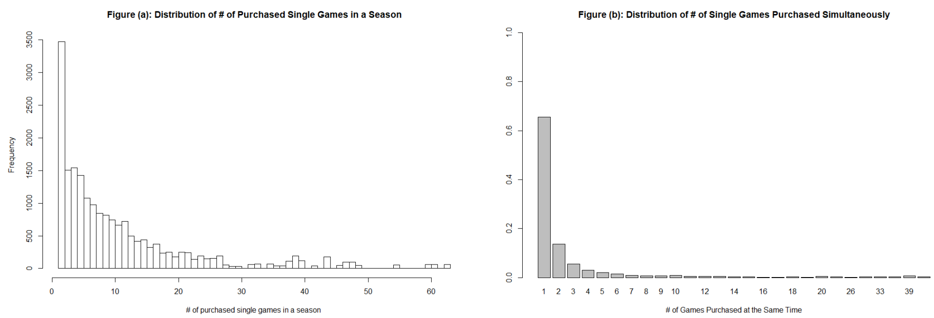


Figure 3: Distribution of the Number of Purchased Single Games

of the total number of purchased games in a season. In general, consumers make purchases on a game-by-game basis. Even when a consumer purchases 15 single games in a season, the majority are purchased on separate game-by-game basis rather than as a joint purchase of multiple games at the same time.

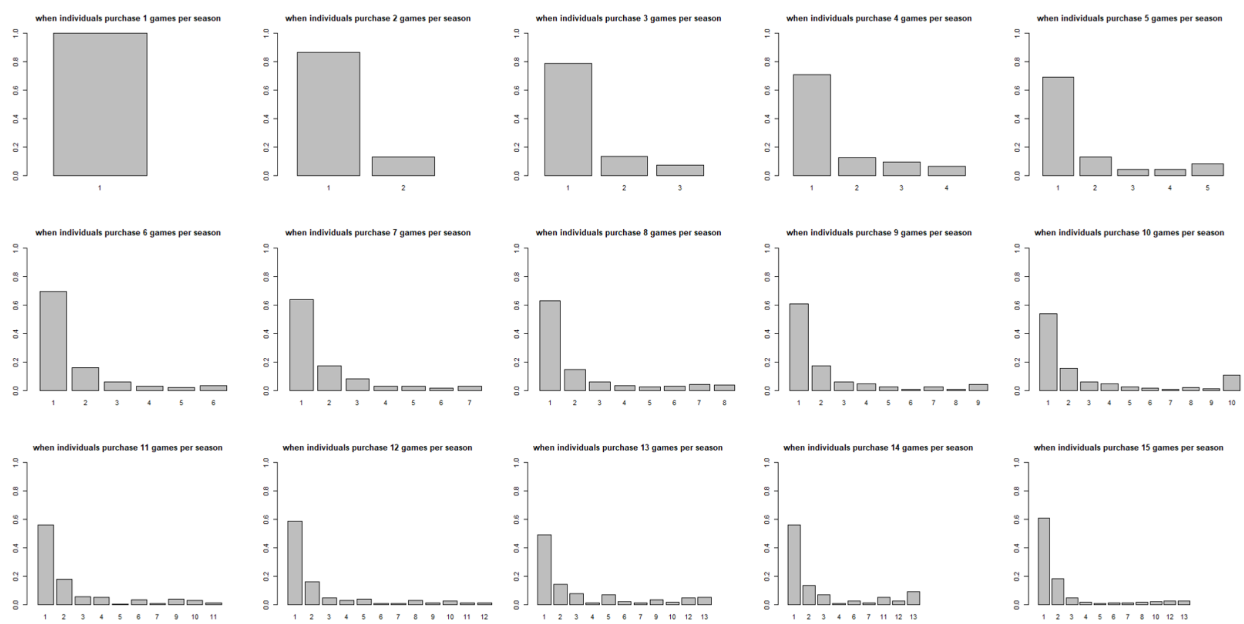


Figure 4: Distribution of Single Game Purchased Simultaneously

Next, we turn to the season ticket purchase decision. In a given season, the team plays 18 different opponents (league members plus interleague opponents). It is possible that season ticket purchasers buy the season ticket packages to attend only a subset of teams. In Figure 5, we plot out the number of games the focal team plays with each of its opponents over the six seasons from 2011 to 2016. Each bar on the x-axis is a unique opponent. We drop the names to keep the focal team anonymous.

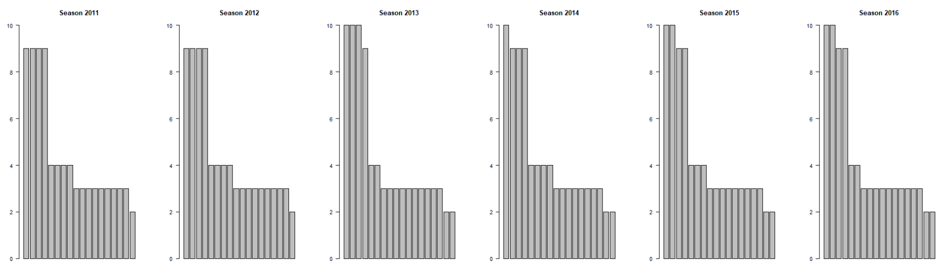


Figure 5: Number of Games with Each Opponent Team

We next construct a Herfindahl index in terms of whether game attendance was concentrated towards certain teams or not. Figure 6 (left) shows the calculated Herfindahl index across all the 1,924 season ticket holders. Almost all the season ticket holders attend games by a number of teams rather than concentrate all their visits to one team. Figure 6 (middle) looks at the relationship between the individual-season level Herfindahl index and the number of overall games attended in a season by the season ticket holder. It appears that regardless of the game attendance rate within a season, season ticket holders prefer to attend a variety of teams rather than only one or two favorite teams. Figure 6 (right) plots the relationship between the variety of teams and the number of games the season ticket holders attended in a season. Again, season ticket holders tend to prefer a wide variety of opponents. Overall, the data suggests that selective bundling of teams may not be a salient concern for season ticket purchase.

Overall, we conducted a variety of model-free analyses related to game bundling decisions. These analyses focus on questions related to whether single game tickets are purchased simultaneously as bundles or sequentially as single games. We also explore whether there are patterns in the sets of games attended by consumers (single game and season ticket buyers). In general, there does not appear to be systematic evidence that consumers are creating their own customized bundles.

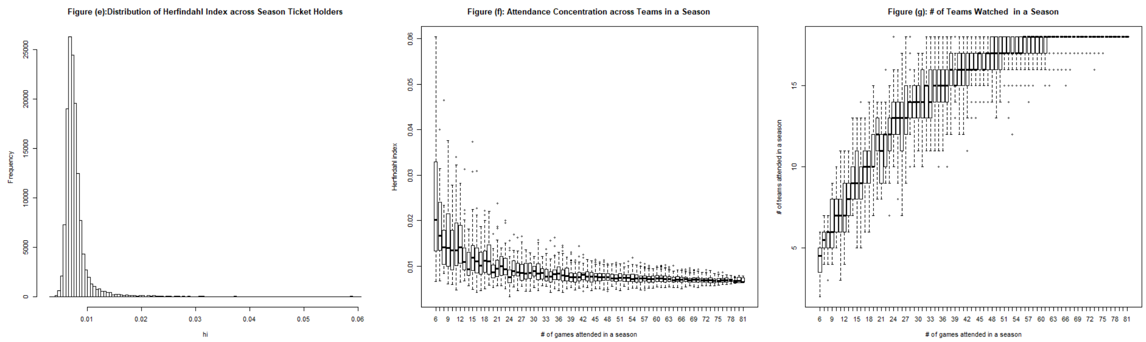


Figure 6: Game Attendance Concentration Across Teams in a Season

B Model Derivation Details

B.1 Optimal List Price Ratio

The following is to show that we can represent the optimal list price ratio using the secondary market demand and supply parameters. We insert the logit expression of $q(r)$ in Equation (5) back to Equation (6) to obtain:

$$r^*_{igt|j} = \frac{\max\{u^A_{igt|j}, u^F_{igt|j}\} - \varepsilon^L_{igt|j}}{\delta \times \beta_{4i} \times GateP_{gt|j}} + \frac{1 + \exp(a_{gt|j} - \gamma_4 r^*_{igt|j})}{\gamma_4}. \tag{B.1}$$

With rearrangement of the above expression, the solution of the optimal list price ratio is to solve the following equation:

$$(a_{gt|j} - \gamma_4 r^*_{igt|j}) + \exp(a_{gt|j} - \gamma_4 r^*_{igt|j}) = a_{gt|j} - \gamma_4 \times \frac{\max\{u^A_{igt|j}, u^F_{igt|j}\} - \varepsilon^L_{igt|j}}{\delta \times \beta_{4i} \times GateP_{gt|j}} - 1. \tag{B.2}$$

If we denote $t_{igt|j} = \exp(a_{gt|j} - \gamma_4 \times \frac{\max\{u^A_{igt|j}, u^F_{igt|j}\} - \varepsilon^L_{igt|j}}{\delta \times \beta_{4i} \times GateP_{gt|j}} - 1)$, the above equation can be re-written as:

$$(a_{gt|j} - \gamma_4 r^*_{igt|j}) + \exp(a_{gt|j} - \gamma_4 r^*_{igt|j}) = \ln t_{igt|j}. \tag{B.3}$$

The solution to this equation can be represented using the Lambert-W function as:

$$a_{gt|j} - \gamma_4 r^*_{igt|j} = \ln t_{igt|j} - W(\exp(\ln t_{igt|j})) = \ln t_{igt|j} - W(t_{igt|j}). \tag{B.4}$$

Equivalently, we have:

$$r^*_{igt|j} = \frac{a_{gt|j} - \ln t_{igt|j} + W(t_{igt|j})}{\gamma_4}, \tag{B.5}$$

and,

$$\exp(a_{gt|j} - \gamma_4 r^*_{igt|j}) = t_{igt|j} \cdot \exp(-W(t_{igt|j})) = W(t_{igt|j}). \tag{B.6}$$

Based on this, we can express the resale probability $q(r^*_{igt|j})$ using Lambert-W function as:

$$q(r^*_{igt|j}) = \frac{\exp(a_{gt|j} - \gamma_4 r^*_{igt|j})}{1 + \exp(a_{gt|j} - \gamma_4 r^*_{igt|j})} = \frac{t_{igt|j} \cdot \exp(-W(t_{igt|j}))}{1 + t_{igt|j} \cdot \exp(-W(t_{igt|j}))} = \frac{W(t_{igt|j})}{1 + W(t_{igt|j})}. \tag{B.7}$$

B.2 The Probability of Listing Season Tickets to Resell

We now look at the probability of season ticket holder i choosing to list a ticket to resell on secondary markets. Season ticket holders will list a ticket on secondary markets if the utility of listing is larger than the maximum utility out of either attending or forgoing a ticket, i.e., $u^L_{igt|j} > \max\{u^A_{igt|j}, u^F_{igt|j}\}$. Specifically, when we insert the utility of listing at $r_{igt|j}$, this condition equals:

$$-c_{it} + q(r_{igt|j}) \cdot (\delta \beta_{4i} \cdot r_{igt|j} \cdot GateP_{gt|j} + \varepsilon^L_{igt|j}) + (1 - q(r_{igt|j})) \cdot \max\{u^A_{igt|j}, u^F_{igt|j}\} > \max\{u^A_{igt|j}, u^F_{igt|j}\}. \tag{B.8}$$

As the error terms in both of the $u^A_{igt|j}$ and $u^F_{igt|j}$ functions follow the standard Type-I extreme value distribution, we have $\max\{u^A_{igt|j}, u^F_{igt|j}\} = v^{AF}_{igt|j} + \varepsilon^{AF}_{igt|j}$, where $v^{AF}_{igt|j} = \ln(\exp(v^A_{igt|j}) + \exp(v^F_{igt|j}))$, and $\varepsilon^{AF}_{igt|j}$ follows the standard Type-I extreme value distribution. We can rewrite the above condition as:

$$\delta\beta_{4i} \cdot GateP_{gt|j} \cdot r_{igt|j} - \frac{c_{it}}{q(r_{igt|j})} > v^{AF}_{igt|j} + \Delta\varepsilon_{igt|j}, \tag{B.9}$$

where $\Delta\varepsilon_{igt|j} = \varepsilon^{AF}_{igt|j} - \varepsilon^{L}_{igt|j}$ follows the standard Type-I extreme value distribution.

The first order condition of $u^L_{igt|j}$ with regard to $r_{igt|j}$ gives us the optimal solution condition as:

$$r^*_{igt|j} = \frac{v^{AF}_{igt|j} + \Delta\varepsilon_{igt|j}}{\delta\beta_{4i} \cdot GateP_{gt|j}} - \frac{q(r^*_{igt|j})}{q'(r^*_{igt|j})}. \tag{B.10}$$

Inserting this optimal condition on $r^*_{igt|j}$ back to the above equation, we have the following equivalence:

$$\delta\beta_{4i} \cdot GateP_{gt|j} \cdot \frac{q(r^*_{igt|j})}{q'(r^*_{igt|j})} + \frac{c_{it}}{q(r^*_{igt|j})} < 0. \tag{B.11}$$

Given our logit demand specification $q(r_{igt|j}) = \frac{e^{a_{jgt} - \gamma_4 r_{igt|j}}}{1+e^{a-\gamma_4 r_{igt|j}}}$, the above reduces to :

$$\frac{\delta\beta_{4i} \cdot GateP_{gt|j}}{\gamma_4} \cdot e^{a_{jgt} - \gamma_4 r^*_{igt|j}} > c_{it}, \tag{B.12}$$

Inserting equation (B.6) into this condition, we have:

$$W(t_{igt|j}) > \frac{c_{it}\gamma_4}{\delta\beta_{4i} \cdot GateP_{gt|j}}. \tag{B.13}$$

Since the Lambert-W function is the inverse function of $f(z) = z\exp(z)$. Applying the inverse function, we have:

$$\ln t_{igt|j} > \frac{c_{it}\gamma_4}{\delta\beta_{4i} \cdot GateP_{gt|j}} + \ln \frac{c_{it}\gamma_4}{\delta\beta_{4i} \cdot GateP_{gt|j}}, \tag{B.14}$$

or equivalently:

$$v^{AF}_{igt|j} + \Delta\varepsilon_{igt|j} < \frac{\delta\beta_{4i} \cdot GateP_{gt|j}}{\gamma_4}(a_{jgt} - 1 - \ln \frac{c_{it}\gamma_4}{\delta\beta_{4i} \cdot GateP_{gt|j}}) - c_{it}. \tag{B.15}$$

The probability of listing a season ticket $\Pr(d^L_{igt|j})$ equals the probability when the above condition satisfies. Given $\Delta\varepsilon_{igt|j}$ follows the standard logistic distribution, this equals:

$$\Pr(d^L_{igt|j}) = \frac{\exp(\phi^L_{igt|j})}{\exp(\phi^L_{igt|j}) + \exp(v^{AF}_{igt|j})} = \frac{\exp(\phi^L_{igt|j})}{\exp(\phi^L_{igt|j}) + \exp(v^{A}_{igt|j}) + \exp(v^{F}_{igt|j})}, \tag{B.16}$$

where $\phi^L_{igt|j} = \frac{\delta\beta_{4i}\cdot GateP_{gt|j}}{\gamma_4}(a_{jgt} - 1 - \ln \frac{\gamma_4 c_{it}}{\delta\beta_{4i}\cdot GateP_{gt|j}}) - c_{it}$. This finishes the derivation for equation (10) in the paper.

B.3 Expected Usage Utility of Season Tickets

The expected overall usage utility for game g is the weighted average of the expected utility of attending or forgoing a ticket $\max\{\tilde{u}^A_{igt|j}, \tilde{u}^F_{igt|j}\}$ and the expected listing utility $\tilde{u}^L_{igt|j}$, with the corresponding weights equal to $\Pr[\max\{\tilde{u}^A_{igt|j}, \tilde{u}^F_{igt|j}\} > \tilde{u}^L_{igt|j}]$ and $\Pr[\max\{\tilde{u}^A_{igt|j}, \tilde{u}^F_{igt|j}\} \leqslant \tilde{u}^L_{igt|j}]$, respectively. We can express the expected usage utility $USE_{igt|j}$ of a season ticket of game g in tier j season t for consumer i as (we absorb subscripts $gt|j$ for simplicity in the derivations):

$$USE_{igt|j} = \Pr\left[\max\{\tilde{u}^A_i, \tilde{u}^F_i\} > \tilde{u}^L_i\right] \cdot \max\{\tilde{u}^A_i, \tilde{u}^F_i\} + \Pr\left[\max\{\tilde{u}^A_i, \tilde{u}^F_i\} \leqslant \tilde{u}^L_i\right] \cdot \tilde{u}^L_i, \tag{B.17}$$

We rewrite $\max\{\tilde{u}^A_i, \tilde{u}^F_i\} = \tilde{v}^{AF}_i + \tilde{\varepsilon}^{AF}_i$ and $\tilde{u}^L_i = \tilde{v}^L_i + \tilde{\varepsilon}^L_i$. Thus, the probability of $\Pr\left[\max\{\tilde{u}^A_i, \tilde{u}^F_i\} > \tilde{u}^L_i\right]$ is equal to $\Pr\left[\tilde{v}^{AF}_i + \tilde{\varepsilon}^{AF}_i > \tilde{v}^L_i + \tilde{\varepsilon}^L_i\right]$, while $\Pr[\max\{\tilde{u}^A_i, \tilde{u}^F_i\} \leqslant \tilde{u}^L_i]$ is equal to $\Pr[\tilde{v}^{AF}_i + \tilde{\varepsilon}^{AF}_i \leqslant \tilde{v}^L_i + \tilde{\varepsilon}^L_i]$. The error terms $\tilde{\varepsilon}^L_i, \tilde{\varepsilon}^{AF}_i$ are both standard Type-I extreme value distributed, such that we have:

$$
\begin{aligned}
USE_{igt|j} =& \int_{-\infty}^{\infty}\int_{\tilde{v}_i^L-\tilde{v}_i^{AF}+\tilde{\varepsilon}_i^L}^{\infty} (\tilde{v}_i^{AF}+\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^L)+ \\
& \int_{-\infty}^{\infty}\int_{-\infty}^{\tilde{v}_i^L-\tilde{v}_i^{AF}+\tilde{\varepsilon}_i^L} -c_{it}+\tilde{q}(r_i^*)(\delta\beta_{4i}r_i^*GateP_{gt|j}+\tilde{\varepsilon}_i^L)+(1-\tilde{q}(r_i^*))(\tilde{v}_i^{AF}+\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^L) \\
=& \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\tilde{v}_i^{AF}+\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^L)+ \\
& \int_{-\infty}^{\infty}\int_{-\infty}^{\tilde{v}_i^L-\tilde{v}_i^{AF}+\tilde{\varepsilon}_i^L} -c_{it}+\tilde{q}(r_i^*)(\delta\beta_{4i}r_i^*GateP_{gt|j}+\tilde{\varepsilon}_i^L-\tilde{v}_i^{AF}-\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^{AF})dF(\tilde{\varepsilon}_i^L),
\end{aligned}
\tag{B.18}
$$

where r_i^* is the optimal price under the secondary market demand function.

The integrals are over $\tilde{\varepsilon}^{AF}$ and $\tilde{\varepsilon}^L$. We can further derive the above expression based on the Type-I extreme value distribution as follows:

$$
\begin{aligned}
USE_{igt|j} =& \tilde{v}_i^{AF}+\lambda+\int_{-\infty}^{\tilde{v}_i^L-\tilde{v}_i^{AF}} -c_{it}+\tilde{q}(r_i^*)(\delta\beta_{4i}r_i^*GateP_{gt|j}-\tilde{v}_i^{AF}-\Delta\tilde{\varepsilon}_i)dF(\Delta\tilde{\varepsilon}_i) \\
=& \tilde{v}_i^{AF}+\lambda+\int_{-\infty}^{\tilde{v}_i^L-\tilde{v}_i^{AF}} -c_{it}+\frac{W(t_i)}{1+W(t_i)}\cdot\frac{\delta\beta_{4i}GateP_{gt|j}}{\gamma_4}[W(t_i)+1]dF(\Delta\tilde{\varepsilon}_i) \\
=& \tilde{v}_i^{AF}+\lambda+\int_{-\infty}^{\tilde{v}_i^L-\tilde{v}_i^{AF}} -c_{it}+\frac{\delta\beta_{4i}GateP_{gt|j}}{\gamma_4}W(t_i)dF(\Delta\tilde{\varepsilon}_i) \\
=& \tilde{v}_i^{AF}+\lambda-c_{it}\cdot\frac{e^{\tilde{v}_i^L}}{e^{\tilde{v}_i^{AF}}+e^{\tilde{v}_i^L}}+\int_{-\infty}^{\tilde{v}_i^L-\tilde{v}_i^{AF}} \frac{\delta\beta_{4i}GateP_{gt|j}}{\gamma_4}W(e^{a-\frac{\gamma_4(\tilde{v}_i^{AF}+\Delta\tilde{\varepsilon}_i)}{\delta\beta_{4i}GateP_{gt|j}}-1})dF(\Delta\tilde{\varepsilon}_i),
\end{aligned}
\tag{B.19}
$$

where λ is the Euler's constant and $t_i = \exp(a_{jgt} - \frac{\gamma_4(\tilde{v}_i^{AF}+\Delta\tilde{\varepsilon}_i)}{\delta\beta_{4i}GateP_{gt|j}} - 1)$, and $\Delta\tilde{\varepsilon}_i = \tilde{\varepsilon}_i^{AF} - \tilde{\varepsilon}_i^L$ follows the standard logistic distribution. We use numerical integration to calculate the last component in the estimation process.

C Likelihood Details

C.1 Likelihood of Purchase and Usage Decisions

We let $\mathcal{L}_{it}(d_{it}, d_{igt}|\Psi, \beta_i, c_i, \{k_i\}, \lambda_i, \tau_i, \gamma)$ be the likelihood of observing consumer i making season ticket purchase choice $d_{it} = \{d^S_{ijt}\}$ for season t, and ticket usage and gate ticket purchase decisions $d_{igt} = \{d^A_{igt|j}, d^F_{igt|j}, d^L_{igt|j}, d^G_{igt}, d^G_{ijgt}, d^{SD}_{igt}, d^{SD}_{ijgt}, d^N_{igt}\}$ for each game g in season t, conditional on observed variables $\Psi = \{X, W, Z, SeasonP, GateP\}$. This forms the likelihood of any given path of ticket purchase and usage choices in season t:

$$\mathcal{L}_{it}(d_{it}, d_{igt}|\Psi, \beta_i, \rho_i, \{k_i\}, \lambda_i, \tau_i, \gamma) = \prod_{j=1}^{6} \left[P^S_{ijt} \times \prod_{g=1}^{81} (P^A_{igt|j})^{d^A_{igt|j}} \times (P^F_{igt|j})^{d^F_{igt|j}} \times (P^L_{igt|j})^{d^L_{igt|j}} \right]^{d^S_{ijt}}$$
$$\times \left[P^S_{i0t} \times \prod_{g=1}^{81} \prod_{j=0}^{6} (P^G_{igt})^{d^G_{igt}} (P^G_{ijgt})^{d^G_{ijgt}} \times (P^{SD}_{igt})^{d^{SD}_{igt}} (P^{SD}_{ijgt})^{d^{SD}_{ijgt}} \times (P^N_{igt})^{d^N_{igt}} \right]^{d^S_{i0t}} . \tag{C.20}$$

We assume the errors ε in Equations (1), (2) and (3) follow standard Type-I extreme value distributions. In the equations below, we use v to denote the deterministic part in each of the utility functions specified above. We write the probability of of a consumer buying a tier j season ticket as:

$$P^S_{ijt} = \frac{e^{v^S_{ijt}}}{\sum_{k=1}^{6} e^{v^S_{ikt}} + e^{v^S_{i0t}}}. \tag{C.21}$$

Similarly, the probability of a consumer not buying a season ticket is:

$$P^S_{i0t} = \frac{e^{v^S_{i0t}}}{\sum_{k=1}^{6} e^{v^S_{ikt}} + e^{v^S_{i0t}}}. \tag{C.22}$$

The probability of a consumer choosing to attend with, forgo, or list a game g ticket conditional on having purchased a tier j season ticket are given, respectively, in equations below:

$$P^A_{igt|j} = \frac{e^{v^A_{igt|j}}}{e^{v^A_{igt|j}} + e^{v^F_{igt|j}} + e^{\phi^L_{igt|j}}}, \tag{C.23}$$

and

$$P^F_{igt|j} = \frac{e^{v^F_{igt|j}}}{e^{v^A_{igt|j}} + e^{v^F_{igt|j}} + e^{\phi^L_{igt|j}}}, \tag{C.24}$$

and

$$P^L_{igt|j} = \frac{e^{\phi^L_{igt|j}}}{e^{v^A_{igt|j}} + e^{v^F_{igt|j}} + e^{\phi^L_{igt|j}}}. \tag{C.25}$$

Alternatively, the probability of a consumer choosing to buy a single ticket from team directly of game g conditional on *not* purchasing a season ticket is:

$$P^G_{igt} = \frac{e^{v^G_{igt}}}{e^{v^G_{igt}} + e^{v^{SD}_{igt}} + e^{v^N_{igt}}}, \tag{C.26}$$

and the probability of a consumer choosing to buy a single ticket from secondary market of game g conditional on *not* purchasing a season ticket is:

$$P^{SD}_{igt} = \frac{e^{v^{SD}_{igt}}}{e^{v^G_{igt}} + e^{v^{SD}_{igt}} + e^{v^N_{igt}}}, \tag{C.27}$$

and the probability of not buying a single ticket of game g would be:

$$P^N_{igt} = \frac{e^{v^N_{igt}}}{e^{v^G_{igt}} + e^{v^{SD}_{igt}} + e^{v^N_{igt}}}. \tag{C.28}$$

The probability of buying tier j single ticket given certain purchase outlet, for example, from the team, would be:

$$P^G_{ijgt} = \frac{e^{v^G_{ijgt}}}{\sum_{k=1}^{6} e^{v^G_{ikgt}}}. \tag{C.29}$$

C.2 Likelihood of Listing Prices

We denote $\mathcal{L}(r_{igt|j}|d^L_{igt|j}, \Psi, \beta_i, \gamma)$ to be the likelihood of observing consumer i listing a game g season ticket at a price ratio of $r_{igt|j}$ on the online secondary market. We have derived the optimal listing price ratio of $r^*_{igt|j}$ in Equation (7) in the paper. The observed listing price ratio may be different from the optimal one from the analytic solution, and we assume the following relationship:

$$r_{igt|j} = r^*_{igt|j} + \varepsilon^r_{igt|j}, \tag{C.30}$$

where $\varepsilon_{igt|j} \sim N(0, \sigma_r^2)$.

We acknowledge that our derivation for the optimal listing ratio is an approximation to the real complex pricing decision, for example, potential potential dynamic pricing patterns are not considered. Because of this, it is more realistic to assume an additional error term when comparing the observed listing price ratio to our model solution. On average, we would expect the observed listing price to be consistent with our derived optimal one, and the variance of the error term reflects overall how close our approximation is.

Based on the formulation, the likelihood can be expressed as (we absorb subscripts $gt|j$ for simplicity):

$$\mathcal{L}_i(r_i|d^L_i, \Psi, \beta_i, \gamma) = \int_{-\infty}^{\phi^L_i - v^{AF}_i} \sigma_r \phi(\frac{r_i - r^*_i(\Delta\varepsilon_i)}{\sigma_r}) f(\Delta\varepsilon_i) d\Delta\varepsilon_i, \tag{C.31}$$

where $\Delta\varepsilon_i = \varepsilon^{AF}_i - \varepsilon^L_i$ is the difference in the error terms for consumer i to either attend or forgo, or list the ticket. As both ε^{AF}_i and ε^L_i are distributed according to the standard Type-I extreme value distribution, $\Delta\varepsilon_i$ follows the standard logistic distribution. $\phi(\cdot)$ is the probability density function (pdf) of the standard normal distribution, and $f(\Delta\varepsilon_i)$ is the standard logistic distribution probability density function. $r^*(\Delta\varepsilon_i)$ is given in Equation (7).

Notice that the integration is not over the entire real line because of sample selection, i.e., we only observe listing price ratios when a customer lists the ticket on the secondary market. In other words, conditional on observing listing, $\Delta\varepsilon_i$ follows a truncated logistic distribution with the truncation given by $\Delta\varepsilon_i < \phi_i^L - v_i^{AF}$. The calculation of this likelihood component involves an integration step. We use numeric integration to simulate a number of random variables from this truncated logistic distribution to calculate this likelihood in the estimation process.

C.3 Likelihood of Secondary Market Sales Outcomes

Next, we denote $\mathcal{L}(y_{k,jgt}|\Psi, L_{jgt}, r_{k,jgt}, \beta, \gamma)$ to be the likelihood of observing a listed ticket k being sold ($y_{k,jgt} = 1$) or not ($y_{k,jgt} = 0$) on the secondary market for game g. Based on the sales probability function $q_{k,jgt}$ as in Equation (4), we have:

$$\mathcal{L}_t(y_{k,jgt}|\Psi, L_{jgt}, r_{k,jgt}, \beta, \gamma) = \prod_{g=1}^{G}\prod_{j=1}^{J}\prod_{k=1}^{K} q_{k,jgt}^{y_{k,jgt}} \times (1 - q_{k,jgt})^{(1-y_{k,jgt})} \tag{C.32}$$

C.4 Overall Log-Likelihood Function

Finally, the three likelihood elements combined form the overall log-likelihood overt T season as:

$$\mathcal{LL}(d_{it}, d_{igt}, r_{igt|j}, y_{k,jgt}) = \sum_{t=1}^{T}\sum_{i=1}^{I} \ln \mathcal{L}_{it}(d_{it}, d_{igt}) + \sum_{t=1}^{T}\sum_{i=1}^{I} \ln \mathcal{L}_{it}(r_{igt|j}) + \sum_{t=1}^{T} \ln \mathcal{L}_t(y_{k,jgt}) \tag{C.33}$$

D Estimation Steps

We use Markov chain Monte Carlo (MCMC) method in our model estimation. We follow the method developed in Yang et al. (2003) and augment the unobserved market shocks ξ_{gt} in the MCMC steps. The detailed estimation process is outlined as follows:

Step 1. Generate γ.

The posterior is: $\pi(\gamma|*) \propto \mathcal{L}(d_{it}, d_{igt}|\theta_i, \gamma, \xi_{gt}) \cdot \mathcal{L}(r_{igt}|\theta_i, \gamma, \xi_{gt}, \sigma_r^2) \cdot \mathcal{L}(y_{kgt}|\theta_i, \gamma, \xi_{gt}) \cdot \pi(\gamma)$. We specify a diffuse prior for γ, $\gamma \sim N(0, V_{\gamma 0})$, and use Metropolis-Hastings algorithm with a random walk chain to generate proposals of γ, i.e, $\gamma_i^{*m} \sim N(\gamma_i^{m-1}, V_\gamma^m)$ at iteration m.

Step 2. Generate ξ_{gt} and Σ_ξ.

The posterior for ξ_{gt} is: $\pi(\xi_{gt}|*) \propto \mathcal{L}(d_{it}, d_{igt}|\theta_i, \gamma, \xi_{gt}) \cdot \mathcal{L}(r_{igt}|\theta_i, \gamma, \xi_{gt}, \sigma_r^2) \cdot \mathcal{L}(y_{kgt}|\theta_i, \gamma, \xi_{gt}) \cdot \pi(\xi_{gt})$. The prior is specified as $\pi(\xi_{gt}) \sim N(0, \Sigma_\xi)$. We use Metropolis-Hastings algorithm with a random walk chain to generate proposals of ξ_{gt}. Based on the current values of ξ_{gt}, we also update the posterior of the variance-covariance matrix Σ_ξ, where $\pi(\Sigma_\xi|\xi_{gt}) \propto N(\xi_{gt}, 0, \Sigma_\xi) \cdot \pi(\Sigma_\xi)$. We assume that the prior for Σ_ξ follows an inverse-Wishart distribution $\Sigma_\xi \sim IW(\nu_0, V_{\xi 0})$. We use Gibbs sampling to update the value given the posterior also follows an inverse-Wishart distribution.

Step 3. Generate (Π, Σ_θ). These two are the hyper-parameters that govern the distribution of $\theta_i \sim N(\Pi D_i, \Sigma_\theta)$. We specify the priors as $\Pi|\Sigma_\theta \sim N(0, \Sigma_\theta \otimes A_\theta^{-1})$ and $\Sigma_\theta \sim IW(\nu_0, V_{\theta 0})$. We use Gibbs sampling to update these two parameters under this multivariate Bayesian regression setting.

Step 4. Generate θ_i.

The posterior for θ_i is: $\pi(\theta_i|*) \propto \mathcal{L}(d_{it}, d_{igt}|\theta_i, \gamma, \xi_{gt}) \cdot \mathcal{L}(r_{igt}|\theta_i, \gamma, \xi_{gt}, \sigma_r^2) \cdot \pi(\theta_i)$. Our hierarchical specification implies that $N(\Pi D_i, \Sigma_\theta)$ is effectively the prior for θ_i. We use Metropolis-Hastings algorithm with a random walk chain to generate proposals of θ_i in this step, i.e., $\theta_i^{*m} \sim N(\theta_i^{m-1}, V_\theta^m)$ at iteration m.

Step 5. Generate σ_r^2.

The posterior for σ_r^2 is: $\pi(\sigma_r^2|r, d_{igt}^L) \propto \int_{r^*} \mathcal{L}(r|r^*) f(r^*|d_{igt}^L) dr^* \cdot \pi(\sigma_r^2)$. We assume the prior of σ_r^2 to follow an inverse-gamma distribution. The likelihood component calculation requires the same numeric integration as specified in Equation (C.31).

We have adopted a few strategies to improve the sampling efficiency in our MCMC estimation iterations. First, we estimated a model with homogeneous θ and without the market shocks ξ_{gt} using the maximum likelihood estimation (MLE) approach. We use the model estimates as the starting values in our MCMC iterations. We also utilize the corresponding Hessian matrices for θ and λ in generating variance matrices (V_θ and V_γ) in the random walk proposals. Second, we scales the variance matrices (V_θ^m and V_γ^m) in the random walk proposals based on the acceptance rates of previous draws to achieve an optimal acceptance rate near 0.23 (Gelman et al., 1996; Roberts et al., 2001). Third, to alleviate the high autocorrelations in the individual parameters θ_i in this hierarchical modeling specification, we adopt the Ancillarity–Sufficiency Interweaving Strategy (ASIS) strategy developed in Yu and Meng (2011). Finally, to speed up the estimation, we utilize the general purpose graphics processing unit (GPGPU) in the likelihood computations. Combining these strategies, our MCMC chains converge well in 100,000 iterations.

E Simulation Study

E.1 Main Model Simulation

In order to check that we can correctly recover the model parameters, we perform a numerical simulation exercise. We simulate 1,000 consumers with heterogeneous preferences making season- and gate-ticket purchase decisions in 3 seat tiers for 5 seasons with 81 games in each season. Conditional on their season ticket purchase decisions, we also simulate their decisions of attending, forgoing and listing for each of the games. In the case of listing season tickets on the secondary market, we also simulate their optimal listing prices, and whether the ticket would result in a successful resale or not. Conditional on a consumer not committing to a season ticket package, we also simulate her decisions in each game, regarding whether to purchase a ticket, which channel (stadium vs. secondary market) to purchase from, and which tier to purchase. Overall, the data simulation steps exactly follow our model setups and generate a data set that is the identical in structure as the real data we use in the model estimation.

The data attributes are simulated according to the following specifications:

- Game attributes X and W: $X \sim N(0, 0.75)$ and $W \sim N(0, 0.5)$.
- Individual demographics D: it is a one-dimensional vector, $D \sim N(0, 1)$.
- Listing cost: we simulate one game-level attribute that affects the listing cost, LC, this is simulated from a standard normal distribution, $LC \sim N(0, 1)$.
- Gate price: gate prices vary across years, with the prices for the tiers to be the following (in USD): year 1: $(88, 60, 42)$; year 2: $(90, 64, 42)$; year 3: $(86, 62, 45)$; year 4: $(86, 64, 35)$; and year 5: $(96, 68, 48)$.
- Price in the secondary market: we assume a consumer can always get a ticket from the secondary market if she does not purchase a season ticket package in advance. The price on the secondary market is assumed to be 0.95 of the normal gate price.

- Season ticket price: season ticket prices are discounted at 20%, 30% and 40%, respectively for the three tiers. The discount rates do not change across years.
- Game shocks ξ: we simulate from a multivariate normal distribution, with mean vector of 0 and variance-covariance matrix of $\Sigma_\xi = \left(\begin{smallmatrix} 0.5 & 0.2 \\ 0.2 & 0.3 \end{smallmatrix}\right)$.
- Secondary market listing commission rate: 0.1, i.e., $\delta = 0.9$.

This simulation process generates an average season ticket purchase rate at 69.1%. Season ticket holders would on average list 9.3% of the tickets in the secondary market. For those who do not purchase season tickets, on average, they would purchase individual tickets at the game stage for 12.2% of the times, and 5.0% of these transactions happen in the online secondary market. These summary statistics generally match with the statistics of our empirical data.

We follow the estimation steps described above to estimate the model parameters from the simulated data. We run 50,000 MCMC iterations in the estimation. The MCMC chains converge well in 20,000 iterations, and we calculate the posterior distributions based on the last 20,000 iterations of the MCMC samples. The true parameters and the mean estimates with their 95% confidence intervals are reported in the following Tables A3 and A4. It is clear from the estimation results that all of our model parameters can be correctly recovered: the mean estimates are very close to the true values, and the true values are contained in the corresponding 95% confidence intervals (with the exception of the mean intercepts for the game attendance utility and for secondary market resale equation, for which the true values are contained in the corresponding 99% confidence intervals).

E.2 Identification On Key Variables

In addition to the main simulation exercise, we also carry out two additional simulations to check the identification of our key model variables. In the first simulation, we set the usage value scalar τ to be 0 and try to recover the model parameters. Note that in our main

Table A3: Model Estimation Results

	True Value	Mean Estimate	2.5 Percentile	97.5 Percentile
Game attendance utility parameters (mean)				
β_{11}: Intercept	1.20	1.31	1.28	1.34
β_{12}: Tier 2	-0.50	-0.50	-0.51	-0.49
β_{13}: Tier 3	-1.00	-1.01	-1.04	-0.99
β_2: X	1.50	1.48	1.41	1.57
β_3: W	2.00	2.03	1.99	2.09
Listing cost				
Intercept	0.50	0.52	0.50	0.54
LC	0.30	0.30	0.29	0.31
Season and gate ticket purchase parameters				
Season purchase intercept	-2.00	-1.95	-2.08	-1.82
Gate purchase intercept	-3.00	-2.98	-3.02	-2.95
Gate secondary market purchase intercept	-3.00	-2.95	-3.01	-2.89
ρ: Nested logit coefficient	-1.50	-1.45	-1.55	-1.40
β_4: Price coefficient	0.60	0.61	0.59	0.63
τ: Season ticket scale	-3.00	-3.06	-3.15	-2.98
Secondary market parameters				
γ_{11}: intercept	1.20	1.53	1.30	1.79
γ_{12}: tier 2	0.60	0.55	0.52	0.60
γ_{13}: tier 3	1.40	1.36	1.30	1.41
γ_2: quality coefficient	0.60	0.62	0.59	0.66
γ_3: listing coefficient	-0.70	-0.63	-0.71	-0.54
γ_4: price coefficient	0.80	0.81	0.79	0.83
Variance parameters				
$\Sigma_\xi(1,1)$	0.50	0.50	0.44	0.58
$\Sigma_\xi(2,2)$	0.30	0.26	0.23	0.30
$\Sigma_\xi(1,2)$	0.20	0.19	0.15	0.24
σ_r^2	0.09	0.09	0.08	0.10
Log-likelihood	-263,341			

Note: The estimates for the individual level parameters are the mean estimates over the population.

estimation, the scalar is transformed to be in the range between 0 and 1, in this exercise we remove this transformation. The estimation result is reported in the top panel in Table A5. From the table, we can find that both the season purchase intercept and the usage value scalar can be correctly identified. All the other parameters are also correctly identified.

In the second simulation, we change the magnitude of the listing cost and re-estimate our model. The identification of listing cost comes from the variations in the proportion of tickets listed. Under our model derivation, if the listing cost is zero or negative, then the consumer will always list the tickets on the secondary market because there is a positive probability

Table A4: Observed Individual Heterogeneity Estimates

	True Value Π	Mean Estimate	2.5 Percentile	97.5 Percentile
Game attendance utility parameters (mean)				
β_{11}: Intercept	0.06	0.06	0.04	0.08
β_{12}: Tier 2	-0.09	-0.10	-0.11	-0.08
β_{13}: Tier 3	0.16	0.18	0.16	0.20
β_2: X	0.02	0.02	0.01	0.04
β_3: W	0.16	0.17	0.15	0.18
Listing cost				
Intercept	0.00	0.00	-0.00	0.01
LC	-0.08	-0.08	-0.09	-0.07
Season and gate ticket purchase parameters				
Season purchase intercept	0.09	0.02	-0.11	0.12
Gate purchase intercept	0.25	0.27	0.22	0.30
Gate secondary market purchase intercept	-0.12	-0.14	-0.21	-0.07
ρ: Nested logit coefficient	0.03	0.12	0.07	0.17
τ: Season ticket scale	0.21	0.21	0.20	0.22
β_4: Price coefficient	0.03	0.06	0.01	0.12

Note: The reported numbers are the coefficient estimates for the individual demographics attribute D. The estimates for the intercepts (mean values) are reported in the main results table.

that she will recoup a value higher than the utility of attending or forgoing the game. On the other hand, if the listing cost is very large, a consumer will never list on the secondary market. Under these two scenarios, because we would not have any data variation, the listing cost parameters can not be identified. In this exercise, we change the intercept of the listing cost to be -4 and 4 respectively in our simulations to check the identification at the ranges where either a very small proportion of listings are observed or a very large proportion of listings are observed in the secondary market. The results reported in Table A5 also confirm that the listing costs can be correctly recovered even when we observe extremely low or high listing actions.

Table A5: Estimation Results When Restricting Key Variables

	True Value	Mean Estimate	2.5 Percentile	97.5 Percentile
Restricting usage value scale $\tau = 0$				
τ	0.00	-0.011	-0.007	0.003
Season purchase intercept	-2.00	-1.90	-2.06	-1.75
Low listing cost scenario				
Listing cost intercept	-4.00	-4.04	-4.10	-3.98
LC	0.30	0.31	0.29	0.32
High listing cost scenario				
Listing cost intercept	4.00	3.97	3.93	4.01
LC	0.30	0.30	0.29	0.31

F Counterfactual Calculations

For simplicity, we omit subscript $igt|j$, $ijgt$ and jgt in all the expressions in this section.

F.1 No Secondary Market

The value of the online secondary market to the sports team from the season ticket holders segment is calculated as the difference between the current observed sales revenue and the expected sales revenue when there is no online secondary market.

In this exercise, we eliminate the online secondary market by removing the additional value brought by the option of listing an ticket on the secondary market. Specifically, we replace equation (13) in the paper with $USE = \max\{\tilde{u}^A, \tilde{u}^F\}$, and use the estimated model parameters to calculate the probability of season ticket purchases for each customer in each game tier in each year and aggregate them to the overall sales revenue. This procedure gives us the estimated economic value of the online secondary market.

F.2 Maximum Resale Price

The impact of restricting the resale price on the secondary market to an upper bound (price ceiling) is calculated as the difference between the current observed sales revenue and the expected sales revenue when such constraints are imposed.

The critical step for this calculation is in setting the equilibrium expected usage utility of season tickets at the purchase stage. We denote the maximum resale price as $\overline{r}$ in the price ratio representation. Equation (13) in the paper now need to satisfy the constraint such that the optimal resale price $r^* \leqslant \overline{r}$. Based on equation (8), this price cap is reached when we have:

$$\frac{a - \ln t + W(t)}{\gamma_4} > \overline{r}, \tag{F.34}$$

which translates to:

$$\tilde{v}^{AF} + \Delta\varepsilon > \delta\beta_4\overline{r} \cdot GateP - \frac{\delta\beta_4 \cdot GateP}{\gamma_4}(1 + e^{a-\lambda_4\overline{r}}). \tag{F.35}$$

We denote the right side as $\overline{C}$ and $\tilde{v}^{\overline{r}} = \delta\beta_4\overline{r} \cdot GateP - c\frac{1+e^{a-\gamma_4\overline{r}}}{e^{a-\gamma_4\overline{r}}}$. Based on the properties of the Type-I extreme value distribution, we can derive the listing probability as:

$$\Pr(d^L) = \begin{cases} \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^L}+e^{\tilde{v}^A}+e^{\tilde{v}^F}}, & \text{if } \tilde{v}^L \leqslant \overline{C} \\ \frac{e^{\tilde{v}^{\overline{r}}}}{e^{\tilde{v}^{\overline{r}}}+e^{\tilde{v}^A}+e^{\tilde{v}^F}}, & \text{if } \tilde{v}^L > \overline{C} \end{cases}. \tag{F.36}$$

The expected overall usage value at the season ticket purchase stage is in the same expression as the previous USE equation when $\tilde{v}^L \leqslant \overline{C}$; the value equals the following when $\tilde{v}^L > \overline{C}$:

$$\begin{aligned} USE =& \tilde{v}^{AF} + \lambda + \int_{-\infty}^{\overline{C}-\tilde{v}^{AF}} -c + W(t) \cdot \frac{\delta\beta_4 GateP}{\gamma_4} dF(\Delta\varepsilon) \\ &+ \int_{\overline{C}-\tilde{v}^{AF}}^{\tilde{v}^L-\tilde{v}^{AF}} -c + \frac{e^{a-\gamma_4\overline{r}}}{1+e^{a-\gamma_4\overline{r}}}(\delta\beta_4\overline{r}GateP - \tilde{v}^{AF} - \Delta\varepsilon) dF(\Delta\varepsilon) \\ =& \tilde{v}^{AF} + \lambda - c \cdot \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^{AF}} + e^{\tilde{v}^L}} + \int_{-\infty}^{\overline{C}-\tilde{v}^{AF}} W(t) \cdot \frac{\delta\beta_4 GateP}{\gamma_4} dF(\Delta\varepsilon) \\ &+ \frac{e^{a-\gamma_4\overline{r}}}{1+e^{a-\gamma_4\overline{r}}}(\delta\beta_4\overline{r}GateP - \tilde{v}^{AF})(\frac{e^{\tilde{v}^L}}{e^{\tilde{v}^L}+e^{\tilde{v}^{AF}}} - \frac{e^{\overline{C}}}{e^{\overline{C}}+e^{\tilde{v}^{AF}}}) \\ &+ \frac{e^{a-\gamma_4\overline{r}}}{1+e^{a-\gamma_4\overline{r}}}\left[\ln(1+e^{\tilde{v}^L-\tilde{v}^{AF}}) - \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^L}+e^{\tilde{v}^{AF}}}(\tilde{v}^L - \tilde{v}^{AF})\right] \\ &- \frac{e^{a-\gamma_4\overline{r}}}{1+e^{a-\gamma_4\overline{r}}}\left[\ln(1+e^{\overline{C}-\tilde{v}^{AF}}) - \frac{e^{\overline{C}}}{e^{\overline{C}}+e^{\tilde{v}^{AF}}}(\overline{C} - \tilde{v}^{AF})\right]. \end{aligned} \tag{F.37}$$

This constraint would also affect the market equilibrium in terms of the probability of listing from each individual season ticket holder, and the secondary market demand function (the resell probability of each individual ticket). For this reason, we also need to numerically calculate the new market equilibrium. The numerical exercise goes through the following steps:

1. For each year, game, seat tier, use the observed listing percentage L_{jgt} as starting point and compute the secondary market demand function regarding a_{jgt}.

2. Iterate the following steps until we get convergence in listing probability L_{jgt} and market demand a.

   (a) Based on the constraint in $\bar{r}$, compute the expected usage value as in equation (F.37), and calculate the season ticket purchase likelihood for each individual customer for each seat tier and each season.

   (b) Compute the listing probability of each season ticket holder according to equation (F.36), and calculate the expected market listing percentage L_{jgt}.

3. Compute the expected season ticket purchase probability according to equation (20) in the paper and compute the expected season ticket sales revenue based on the above converged values.

F.3 Minimum Resale Price

The counterfactual exercise for the minimum resale price follows the same steps as outlined above for the maximum resale price case. The only differences are in terms of the calculations of listing probability and expected overall usage value for season tickets. We derive these formulas below.

We denote the minimum resale price as $\underline{r}$ in the price ratio representation. Equation (13) in the paper now need to satisfy the constraint such that the optimal resale price $r^* \geqslant \underline{r}$.

Based on equation (8), this price cap is reached when we have:

$$\frac{a - \ln t + W(t)}{\gamma_4} < \underline{r}, \tag{F.38}$$

which translates to:

$$\tilde{v}^{AF} + \Delta\varepsilon < \delta\beta_4\underline{r} \cdot GateP - \frac{\delta\beta_4 \cdot GateP}{\gamma_4}(1 + e^{a-\lambda_4\underline{r}}). \tag{F.39}$$

We denote the right side as $\underline{C}$ and $\tilde{v}^{\underline{r}} = \delta\beta_4\underline{r} \cdot GateP - c\frac{1+e^{a-\gamma_4\underline{r}}}{e^{a-\gamma_4\underline{r}}}$. Based on the properties of the Type-I extreme value distribution, we can derive the listing probability as:

$$\Pr(d^L) = \begin{cases} \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^L}+e^{\tilde{v}^A}+e^{\tilde{v}^F}}, & \text{if } \tilde{v}^L \geqslant \underline{C} \\ \frac{e^{\tilde{v}^{\underline{r}}}}{e^{\tilde{v}^{\underline{r}}}+e^{\tilde{v}^A}+e^{\tilde{v}^F}}, & \text{if } \tilde{v}^L < \underline{C} \end{cases}. \tag{F.40}$$

The expected overall usage value at the season ticket purchase stage equals the following when $\tilde{v}^L > \underline{C}$:

$$\begin{aligned} USE =& \tilde{v}^{AF} + \lambda + \int_{-\infty}^{\underline{C}-\tilde{v}^{AF}} -c + \frac{e^{a-\gamma_4\underline{r}}}{1+e^{a-\gamma_4\underline{r}}}(\delta\beta_4\underline{r}GateP - \tilde{v}^{AF} - \Delta\varepsilon)dF(\Delta\varepsilon) \\ &+ \int_{\underline{C}-\tilde{v}^{AF}}^{\tilde{v}^L-\tilde{v}^{AF}} -c + W(t) \cdot \frac{\delta\beta_4 GateP}{\gamma_4} dF(\Delta\varepsilon) \\ =& \tilde{v}^{AF} + \lambda - c \cdot \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^{AF}} + e^{\tilde{v}^L}} + \frac{e^{a-\gamma_4\underline{r}}}{1+e^{a-\gamma_4\underline{r}}}(\delta\beta_4\underline{r}GateP - \tilde{v}^{AF})\frac{e^{\underline{C}}}{e^{\underline{C}} + e^{\tilde{v}^{AF}}} \\ &+ \frac{e^{a-\gamma_4\underline{r}}}{1+e^{a-\gamma_4\underline{r}}}\left[\ln(1+e^{\underline{C}-\tilde{v}^{AF}}) - \frac{e^{\underline{C}}}{e^{\underline{C}}+e^{\tilde{v}^{AF}}}(\underline{C} - \tilde{v}^{AF})\right] \\ &+ \int_{\underline{C}-\tilde{v}^{AF}}^{\tilde{v}^L-\tilde{v}^{AF}} W(t) \cdot \frac{\delta\beta_4 GateP}{\gamma_4} dF(\Delta\varepsilon). \end{aligned} \tag{F.41}$$

When $\tilde{v}^L \leqslant \underline{C}$, we would have the following:

$$\begin{aligned}
USE =&\tilde{v}^{AF} + \lambda + \int_{-\infty}^{\tilde{v}^L - \tilde{v}^{AF}} -c + \frac{e^{a-\gamma_4 \underline{r}}}{1+e^{a-\gamma_4 \underline{r}}}(\delta\beta_4 \underline{r} GateP - \tilde{v}^{AF} - \Delta\varepsilon) dF(\Delta\varepsilon) \\
=&\tilde{v}^{AF} + \lambda - c \cdot \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^{AF}} + e^{\tilde{v}^L}} + \frac{e^{a-\gamma_4 \underline{r}}}{1+e^{a-\gamma_4 \underline{r}}}(\delta\beta_4 \underline{r} GateP - \tilde{v}^{AF}) \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^L} + e^{\tilde{v}^{AF}}} \\
&+ \frac{e^{a-\gamma_4 \underline{r}}}{1+e^{a-\gamma_4 \underline{r}}} \left[\ln(1 + e^{\tilde{v}^L - \tilde{v}^{AF}}) - \frac{e^{\tilde{v}^L}}{e^{\tilde{v}^L} + e^{\tilde{v}^{AF}}} (\tilde{v}^L - \tilde{v}^{AF}) \right].
\end{aligned} \tag{F.42}$$

F.4 Listing Cost Change

The impact of changing the secondary market listing cost is calculated as the difference between the current sales revenue and the expected revenue when such changes are imposed.

Changing the listing cost would also change the equilibrium condition regarding each individual's listing probability and the secondary market demand function (the resell probability of each individual ticket). The counterfactual exercise for the scenario with a different level of listing cost follows the steps below:

1. For each year, game, seat tier, use the observed listing percentage L_{jgt} as starting point and compute the secondary market demand function regarding a_{jgt}.
2. Iterate the following steps until we get convergence in listing probability L_{jgt} and market demand a.
    (a) Based on each individual's new listing cost, compute the expected usage value as in equation (14) in the paper, and calculate the season ticket purchase likelihood for each individual customer for each seat tier and each season.
    (b) Compute the listing probability of each season ticket holder according to equation (11), and calculate the expected market listing percentage L_{jgt}.
3. Compute the expected season ticket purchase probability according to equation (20) in the paper and compute the expected season ticket sales revenue based on the above converged values.

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